A 4D IIB Flux Vacuum with SUSY Breaking and Boundaries

Based on J.M, A. Sagnotti, J.Phys.A 57 (2024) 3, 035401 [2309.05268]; JHEP 11 (2023) 061 [2309.04026]; JHEP 08 (2023) 041 [2305.09587] ; JHEP 08 (2022) 301 [2206.03340]

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April 2025

- Non–Supersymmetric 10D strings have **STABLE vacua** with broken SUSY, with **internal intervals**, with **strong–coupling regions & singular ends**.
- They also have **UNSTABLE** vacua **without strong–coupling or singular regions**.

> **HERE :** IIB vacua with **broken SUSY**
> - **NO STRONG COUPLING regions**
> - **BUT:** internal intervals with **singular ends**

- In compactifications to Minkowski space, perturbative **stability** is seen from the **signs of m^2 for bosonic modes**.

- **The Background:** IIB with a self dual flux Φ on T_5 dependent on one coordinate. Finite interval ℓ.
- **Probe brane:** effective orientifold on one boundary.
- **Broken SUSY: Half of SUSY** is recovered in the $\ell \to \infty$ limit.
- The modes and their **stability** ($\mathbf{m^2 \geq 0}$) depend on singular potentials. Boundary conditions are constrained by **self-adjointness**. Stable boundary conditions exist.
- 4D **massless gravitinos** and spin $1/2$ modes (at tree-level) possible.

- **IIB solution :**

$$ds^2 = e^{2A(r)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2B(r)}dr^2 + e^{2C(r)}dy^2$$

$$e^{2A} = F(r)^{-1}, \quad e^{2B} = F(r)\, e^{-\frac{\sqrt{10}}{2\rho}r}, \quad e^{2C} = F(r)\, e^{-\frac{\sqrt{10}}{10\rho}\, r}$$

$$F(r) = \left[2\,|H|\,\rho\,\sinh\left(\frac{r}{\rho}\right)\right]^{\frac{1}{2}}, \quad \varphi = 0$$

$$\mathcal{H}_5 = H\left\{\frac{dx^0\wedge\ldots\wedge dx^3\wedge dr}{F(r)^4} + dy^1\wedge\ldots\wedge dy^5\right\}$$

NO STRONG COUPLING REGIONS

$$\left(0 \;\le\; y^i \;\le\; 2\,\pi R\right)$$

Boundaries : at $r = 0, \infty$

$$\boxed{\begin{gathered}\int_0^\infty e^B\, dr \sim \ell = (H\rho)^{\frac{1}{4}}\rho < \ \infty \\ \int e^{B-A} dr = \int dz = z_m \sim \ell (H\rho)^{\frac{1}{4}} < \infty\end{gathered}}$$

- Boundaries at **finite distance** and **finite conformal distance**.
- **Singularities :** $z = 0, z_m$.
- **Internal flux :** $\Phi \sim HR^5$
- **Volume of the six extra dimensions :**

$$V_6 \ \sim \ \Phi\, \ell^2 \ \sim \ H^{\frac{3}{2}}\, \rho^{\frac{5}{2}}\, R^5 \ ,$$

- **Volume of internal torus**, which can be defined as $\frac{V_6}{\ell}$,

$$V_5 \ \sim \ \Phi\, \ell \ \sim \ H^{\frac{5}{4}}\, \rho^{\frac{5}{4}}\, R^5 \ .$$

The r=0 boundary and the SUSY solution

- The $\rho \to \infty$ limit preserves **half of the 10D susy**, within a curved spacetime that still includes the singularity at $r = 0$.
- Letting $\xi\, H \;=\; \frac{2}{5}\,(2\,H\,r)^{\frac{5}{4}}$ the solution is

$$ds^2 = \frac{\eta_{\mu\nu}\, dx^\mu\, dx^\nu}{\left(\frac{5}{2}H\xi\right)^{\frac{2}{5}}} + d\,\xi^2 + \left(\frac{5}{2}H\xi\right)^{\frac{2}{5}} \left(d\,y^i\right)^2$$

$$\mathcal{H}_5 = H \left\{ \frac{dx^0 \wedge \ldots \wedge dx^3 \wedge d\xi}{\left(\frac{5}{2}\,H\,\xi\right)^{\frac{9}{5}}} \;+\; dy^1 \wedge \ldots \wedge dy^5 \right\}$$

- The volumes V_6 and V_5 are now **infinite**.
- The finite–ρ solution has **no killing spinors**.
- The finite–ρ and $\rho = \infty$ solutions **coincide** near $r = 0$.

A Probe brane and the effective BPS orientifold

The effective Lagrangian for a **probe** $D3$–brane spanning 4D, with fixed internal coordinates and an r **coordinate that evolves in time**, is determined by the induced metric and the coupling to the gauge field

$$\mathcal{H}_5 \;=\; (1 \;+\; \star)\; dx^0 \wedge \ldots \wedge dx^3 \wedge b'(r)\, dr \;.$$

from

$$\boxed{\frac{\mathcal{S}}{V_3} = -\,T_3 \int\; dt\, e^{4A(r(t))}\, \sqrt{1 \,-\, e^{2(B-A)(r(t))}\, \dot{r}(t)^2} + 2q_3 \int b[r(t)]\; dt}$$

where T_3 is the brane tension and q_3 is the brane charge.

$$b'(r) = \frac{H}{F^4} = \frac{1}{4\,H}\,\frac{1}{\left[\rho\,\sinh\left(\frac{r}{\rho}\right)\right]^2}\;,$$

Energy conservation condition:

$$\boxed{\frac{T_3\ e^{4A(r(t))}}{\sqrt{1\ -\ e^{2(3A+5C)(r(t))}\,\dot{r}(t)^2}}\ -\ 2\,q_3\,b = E}$$

Close to $r = 0$ the limiting behavior of the background is **universal**, and in the non–relativistic limit the equation becomes

$$\frac{T_3}{2}\,\dot{r}^2\ +\ T_3\,e^{4A}\ -\ 2\,q_3\,b\ = E\ ,$$

from which one can identify, near $r \sim 0$, the potential

$$\boxed{V \sim\ \frac{1}{r}\left[T_3\ +\ q_3\,sign(H)\right]}$$

A **gravitational repulsion** sized by the T_3 term and an **"eletric" interaction**, repulsive for $q_3 H > 0$ and attractive for $q_3 H < 0$

Bosonic modes and stability

- The linearized perturbations of the background

$$\boxed{\begin{gathered} ds^2 = e^{2\Omega(z)}\left[(\eta_{\mu\nu} + e^{ik.x} h_{\mu\nu})dx^\mu\, dx^\nu + (1 + e^{ik.x} f(z))dz^2\right] \\ + e^{2C}(\delta_{ij} + e^{ik.x} h_{ij})dy^i dy^j \ , \ \varphi \ = \ e^{ik.x}\varphi(z) \ , \ldots \end{gathered}}$$

with $k^2 = -m^2$, obey **Schrödinger-like equations**

$$\boxed{H\psi = \ m^2\psi, \qquad \text{with} \qquad H \ = \ -\,\partial_z^2 \ + \ V(z)}$$

- The lowest eigenvalue determines the effective 4D physics: **stability and range** of the modes

- The Schrödinger potentials for the different perturbations develop **double–pole singularities at the ends of the interval**, and in their neighborhood they behave as

$$\boxed{V \sim \frac{\mu^2 - \frac{1}{4}}{z^2}\,, \qquad V \sim \frac{\tilde{\mu}^2 - \frac{1}{4}}{(z_m - z)^2}}$$

Case	Sector	μ	$\tilde{\mu}$
1	$\varphi,\ a,\ h_{\mu\nu},\ h_{ij}$	$\frac{1}{3}$	0
2	$B_{\mu\nu}$	$\frac{2}{3}$	1.72
2	${b_{\mu\nu}}^{ij}$	$\frac{1}{3}$	1.09
2	V_μ	$\frac{1}{3}$	2.18
2	$h_{\mu i}$	$\frac{2}{3}$	1.18
2	ϕ_i	$\frac{2}{3}$	2.27
2	ϕ	$\frac{2}{3}$	1
3	$B_{\mu i}$	$\frac{1}{3}$	0.54
3	B_{ij}	$\frac{2}{3}$	0.63
3	${b_\mu}^{ij}$	$\frac{2}{3}$	0.09

Boundary conditions

- The Hamiltonian with domain $\mathcal{D}$ should be **self-adjoint** so that the spectrum be **complete** :

> $(\psi|H\chi) = (H\psi|\chi)$ $(*)\,\forall\psi,\chi \in \mathcal{D}$ **and** if $(*)$ holds $\forall\chi \in \mathcal{D}$ then $\psi \in \mathcal{D}$.

- H with such **boundary conditions** defines a **self-adjoint extension**.
- **Example 1 :** $p = -i\partial$ on $[0, z_m]$ and $\mathcal{D} : \psi(0) = 0 = \psi(z_m)$. (*) holds for all ψ and χ in $\mathcal{D}$, p is symmetric. But the domain of $p^\dagger$ is bigger than $\mathcal{D}$ so p is **not self-adjoint**.
- if the domain of p is $\mathcal{D}_\theta : \psi(0) = e^{i\theta}\psi(z_m)$ then the domain of $p^\dagger$ is $\mathcal{D}_\theta$ and p is **self-adjoint**.

- The spectrum depends on the self-adjoint extension.
- **Example 2 :** $H = -\partial^2$ on $[0, \infty[$ condition (*) gives

$$\psi^*(0)\partial\chi(0) - \chi(0)\partial\psi^*(0) = 0 = W(\psi, \chi)(0).$$

- If $\mathcal{D}_\alpha : \psi'(0) = \alpha\psi(0)$ and $H_\alpha = -\partial^2$ has domain $\mathcal{D}_\alpha$ then $H_\alpha^\dagger = H_{\alpha^*}$. So H_α is **self-adjoint** for real α.
- The spectrum of H_α has one negative eigenvalue $(-\alpha^2)$ if $\alpha < 0$ with eigenvector $\sim e^{\alpha x}$.

Self-Adjoint extensions of the singular Hamiltonians

- the self–adjoint extensions of $H = -\partial^2 + V$ with $V \sim \frac{\mu^2 - \frac{1}{4}}{z^2}$, $V \sim \frac{\tilde{\mu}^2 - \frac{1}{4}}{(z_m - z)^2}$ can be characterized via the **limiting behavior** of the wavefunctions at the two ends :

$$
\begin{aligned}
\psi &\sim C_1 \left(\frac{z}{z_m}\right)^{\frac{1}{2} - \mu} + C_2 \left(\frac{z}{z_m}\right)^{\frac{1}{2} + \mu} \text{if } 0 < \mu < 1 \\
\psi &\sim C_1 \left(\frac{z}{z_m}\right)^{\frac{1}{2}} \log\left(\frac{z}{z_m}\right) + C_2 \left(\frac{z}{z_m}\right)^{\frac{1}{2}} \text{ if } \mu = 0 \\
\psi &\sim C_3 \left(1 - \frac{z}{z_m}\right)^{\frac{1}{2} - \tilde{\mu}} + C_4 \left(1 - \frac{z}{z_m}\right)^{\frac{1}{2} + \tilde{\mu}} \text{if } 0 < \tilde{\mu} < 1 \\
\psi &\sim C_3 \left(1 - \frac{z}{z_m}\right)^{\frac{1}{2}} \log\left(1 - \frac{z}{z_m}\right) + C_4 \left(1 - \frac{z}{z_m}\right)^{\frac{1}{2}} \text{if } \tilde{\mu} = 0
\end{aligned}
$$

- The "local " **extensions** are given by fixing C_1/C_2 and C_3/C_4.
- If $\mu \geq 1$ only $z^{\frac{1}{2} + \mu}$ is in L^2: **unique self-adjoint extension**.

The dilaton & graviton Schrödinger operator

- $H = \mathcal{A}\,\mathcal{A}^\dagger$, $\mathcal{A} = \partial_z + \frac{1}{2}\left(3\,A_z\ +\ 5\,C_z\right)$, $\mathcal{A}^\dagger = -\partial_z + \frac{1}{2}\left(3\,A_z\ +\ 5\,C_z\right)$
- $\mathcal{A}^\dagger\,g\ =\ 0$, gives a **normalizable zero mode** $\boxed{g\ =\ g_0\ e^{\frac{3A+5C}{2}}}$
- **Limiting behavior** of the zero–mode wave-function near the ends:

$$\boxed{g \sim \left(1\ -\ \frac{z}{z_m}\right)^{\frac{1}{2}}\ , g\ \simeq\ \left(\frac{z}{z_m}\right)^{\frac{1}{6}}\ -\ 0.71\left(\frac{z}{z_m}\right)^{\frac{5}{6}}}$$

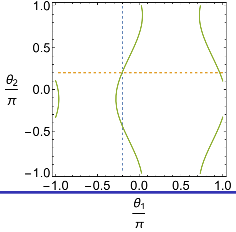
- These limiting behaviors define the **self–adjoint boundary conditions** characterizing the zero mode. This extension of the Hamiltonian leads to **no tachyons: STABILITY**

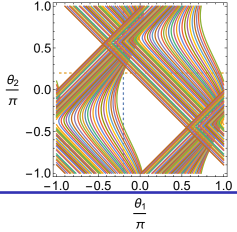
Stability

- More general boundary conditions can be explored relying on the exactly solvable model potential

$$\boxed{V \simeq \frac{\pi^2}{4\, z_m^2} \left[\frac{-\frac{5}{36}}{\sin^2\left(\frac{\pi\, z}{2\, z_m}\right)} - \frac{\frac{1}{4}}{\cos^2\left(\frac{\pi\, z}{2\, z_m}\right)} \right] + \frac{\pi^2}{z_m^2}\, a^2}$$

- Other boundary conditions also lead to **massless modes** (left, below). The **Stability region** (right, below) can be parametrized by $C_1/C_2 = \tan[(\theta_1 - \theta_2)/2]$, $C_3/C_4 = \tan[(\theta_1 + \theta_2)/2]$





Hypergeometric potentials

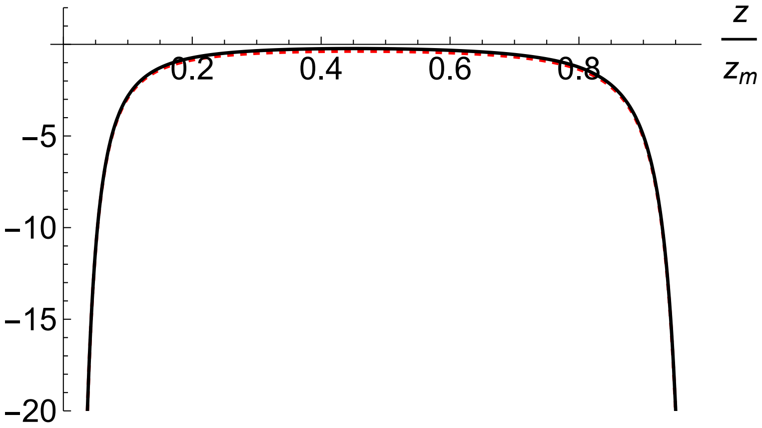


Figure: A comparison between the actual Schrödinger potential for the dilaton and graviton modes (black, solid) and the corresponding hypergeometric approximation (red, dashed).

- The equation of motion of the perturbation ψ can be obtained from the action

$$\boxed{S = \langle \psi | (H - m^2) \psi \rangle,}$$

- with ψ, $H\psi$ in $L^2[0, z_m]$ and H **self-adjoint**.
- The domain of H leads to generalize the "boundary conditions" in terms of C_1/C_2 and not in terms of $\psi(0), \psi'(0)$ which can be not defined.
- For spin 2 the self-adjoint action is **intermediate** between the Einstein-Hilbert and Einstein-Hilbert + York-Gibbons-Hawking.

Fermionic modes

- A doublet of Majorana-Weyl gravitini ψ_M and dilatini λ.
- Equations of motion

$$\boxed{\begin{aligned} &\Gamma^{MNP}\, D_N\, \psi_P \;+\; \frac{1}{8}\, \Gamma^{[M}\, \not{H}\, \Gamma^{N]}\, i\, \sigma_2\, \psi_N \;=\; 0 \\ &\Gamma^{M}\, D_M\, \lambda \;+\; \frac{1}{4}\, \not{H}\, i\, \sigma_2\, \lambda \;=\; 0 \end{aligned}}$$

- The 4D spin $3/2$ modes Ξ_μ:

$$\psi_\mu^{\pm}(x,z) \;=\; \Xi_\mu^{\pm}(x)\, g^{\pm}(z)\, e^{-A-\frac{5}{2}C}\,,\ \Lambda\psi_\mu^{\pm} = \pm\psi_\mu^{\pm}$$

- $\Lambda = \gamma^{0123}\, i\, \sigma_2$ with

$$\not{\partial}\, \Xi_\mu^{\pm}(x) \;=\; \pm\, m\, \gamma_r\, \Xi_\mu^{\mp}(x),$$

a mass m spin $3/2$ mode $\gamma^\mu \Xi_\mu^{\pm} = 0$.

Spin–3/2 modes

The equations for the profile reduce to

$$\boxed{\mathcal{A}\, g^{-} = m\, g^{+}\ , \qquad \mathcal{A}^{\dagger}\, g^{+}\ = m\, g^{-}}$$

where

$$\mathcal{A} \ = \ \partial_z + \mathcal{W}\ ,\ \mathcal{A}^{\dagger} = -\,\partial_z + \mathcal{W}\ . \qquad \mathcal{W} = \frac{H}{2}\ e^{A-5C}$$

$$\mathcal{W}_{z\sim 0} \ \sim \ \frac{1}{6z},\ \mathcal{W}_{z\sim z_m} \sim 0. \quad .$$

we have

$$\mathcal{A}\mathcal{A}^{\dagger} g^{+} = m^2 g^{+}, \quad \mathcal{A}^{\dagger}\mathcal{A} g^{-} = m^2 g^{-},$$

with $\mu_+ = 1/3, \tilde{\mu}_+ = 1/2,\ \mu_- = 2/3,\ \tilde{\mu}_- = 1/2$

Spin–3/2 zero modes

- The gravitino spectrum and z profile are determined by

$$\boxed{Qg = m^2 g \ , \quad g = \begin{pmatrix} g^+ \\ g^- \end{pmatrix} \ , \ Q = \begin{pmatrix} 0 & \mathcal{A} \\ \mathcal{A}^\dagger & 0 \end{pmatrix}}$$

- with

$$\boxed{g = \begin{pmatrix} C_1 z^{\frac{1}{6}} \\ C_2 z^{-\frac{1}{6}} \end{pmatrix} \ , \ z \sim 0, \qquad g = \begin{pmatrix} C_3 \\ C_4 \end{pmatrix} \ , \ z \sim z_m}$$

- Q is self-adjoint if

$$\boxed{C_2 = \gamma_0 C_1 \ , \quad C_4 = \gamma_m C_3}$$

- A pair (γ_0, γ_m) defines a **self-adjoint extension** of Q

Spin–3/2 zero modes

- The solution to $Qg = 0$ given by

$$\boxed{g^{+}(z) = g_0 e^{\int \mathcal{W}\, dz} = g_0 \left[2\,\rho \tanh\left(\frac{r}{2\rho}\right)\right]^{\frac{1}{4}}, \quad g^{-}(z) = 0}$$

$(C_2 = C_4 = 0)$: eigenvector of self-adjoint extension of Q with $(\gamma_0, \gamma_m) = (0, 0)$. Similarily

$$\boxed{g^{+}(z) = 0, \quad g^{-}(z) = g_0 e^{-\int \mathcal{W}\, dz} = g_0 \left[2\,\rho \tanh\left(\frac{r}{2\rho}\right)\right]^{-\frac{1}{4}}}$$

$(C_1 = C_3 = 0)$: eigenvector for $(\gamma_0, \gamma_m) = (\infty, \infty)$

- $\exists$ a massless gravitino mode for both self-adjoint extensions
- Spin 1/2 modes from $\lambda, \psi_i, \gamma^i \psi_i = 0$: same operator Q

Conclusion

- A **non susy 4D** vacuum of IIB with **finite string coupling**
- But **singularities** $\rightarrow$ boundary conditions $\rightarrow$ **self-adjoint** perturbation Hamiltonians
- Self-adjointness $\longleftrightarrow$ **complete spectra**
- **Stability:** can be obtained also with massless fermions

Many possible boundary conditions: Any constraints ?

Thank you