On the stability of non-supersymmetric string vacua

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mainly based on work in progress with V. Menet

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Introduction

- A lot of progress in supersymmetric solutions, especially with geometric methods

  It would be nice to use this wisdom for supersymmetry breaking as well

- In fact most non-susy AdS solutions are unstable or metastable

  We don't live in AdS, but learning to control these effects seems a useful exercise;

  it would be useful to have non-susy holography

- This talk: geometrical ideas to look for non-susy AdS vacua

  that are promising from a stability point of view

Plan

- Review: supersymmetry and stability
    - ‘Calibrated’ non-supersymmetric vacua
- Old and new examples: Kähler–Einstein, homogeneous

Susy & stability

Recall: supersymmetry can be reformulated in terms of forms

- $M_{10} = \mathrm{AdS}_4 \times M_6$

  $\mathrm{e}^{2A} \mathrm{d}s_4^2 + \mathrm{d}s_6^2$

  warping

type II: $$\epsilon^1 = \zeta_+^1 \otimes \eta_+^1 + \text{conj.}$$ $$\epsilon^2 = \zeta_+^2 \otimes \eta_\mp^2 + \text{conj.}$$

$\eta_- = (\eta_+)^c$

- RR$\neq 0$ $\Rightarrow$ $\eta^a \neq 0$ can be encoded as pair (Φ_1, Φ_2) of *compatible pure forms*

- Ex.: $\eta^1 \propto \eta^2$ $\Rightarrow$ $\Phi_1 = \Omega$, $\Phi_2 = \mathrm{e}^{\mathrm{i}\theta} \mathrm{e}^{-\mathrm{i}J}$

concretely

$$\Omega_{mnp} = -\eta_-^\dagger \gamma_{mnp} \eta_+$$
$$J_{mn} = -\mathrm{i}\, \eta_+^\dagger \gamma_{mn} \eta_+$$

- *Almost calibration* property: $|\mathrm{Re}\Phi|_\Sigma| \leqslant \mathrm{vol}_\Sigma$

pure spinor equations:

supersymmetry $\Longleftrightarrow$ $\left\{\begin{array}{l} [d_H(e^{A-\phi}\mathrm{Re}\Phi_2)=0] \\ d_H(e^{2A-\phi}\Phi_1)=\frac{2}{L}e^{A-\phi}\mathrm{Re}\Phi_2 \\ d_H(e^{3A-\phi}\mathrm{Im}\Phi_2)=\frac{3}{L}e^{2A-\phi}\mathrm{Im}\Phi_1-e^{4A}*\lambda f \end{array}\right.$

D-term

$\partial_{\Phi_2}W=0$

$\partial_{\Phi_1}W=0$

[Graña, Minasian, Petrini, AT '05]

NSNS

AdS radius

internal RR

$\lambda(\alpha_k)\equiv(-)^{\lfloor k/2\rfloor}\alpha_k$

$d_H\equiv d-H\wedge$

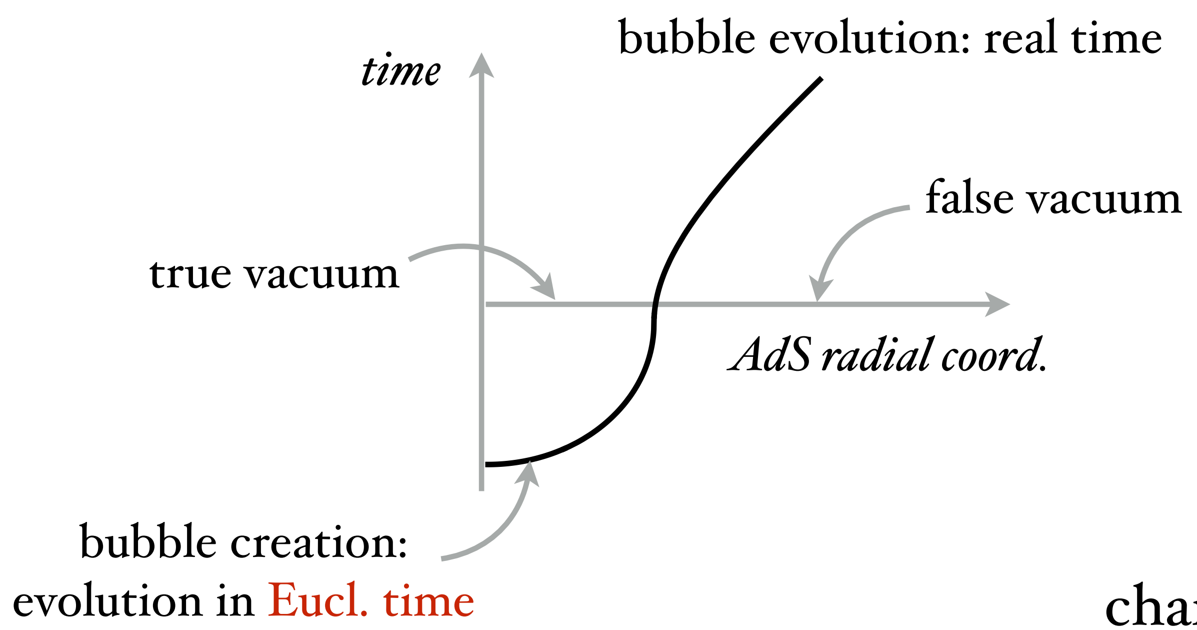
- $\Lambda=0$: M_6 is *generalized complex* [not important today]

- each equation has a 4d interpretation [Koerber, Martucci '07]

- calibrated subspaces are now energy minimizers for brane probes [Martucci, Smyth '05]

- Quick argument to see stability under *brane bubbles* [Giri, Martucci, AT '22]

An unstable vacuum can decay via a bubble of true vacuum



[Coleman, De Luccia '80]
[Maldacena, Michelson, Strominger '98]

the bubble:

$$\frac{\text{charge density}}{\text{tension}} \equiv \frac{q}{\tau} \begin{cases} < 1: \text{ collapses} \\ = 1: \text{ static} \\ > 1: \text{ expands} \end{cases}$$

- same condition for the bubble to nucleate in the first place.

- by similarity with weak gravity conjecture: maybe a bubble with $q/\tau \geqslant 1$ always exists?

[Ooguri, Vafa '16, Freivogel, Kleban '16]

- Indeed an expanding bubble has been found for most AdS vacua. Often it's in fact a D-brane

- Some exceptions have already been proposed

[Córdova, De Luca, AT '18; Giambrone, Guarino, Malek, Samtleben, Sterckx, Trigiante '21]

- Susy vacua are stable: positivity theorems [Witten '81; Abbott, Deser '82, Gibbons, Hull, Warner '83, Hull '83]

adapted to d=10,11 in [Kowalski-Glikman '85, Giri, Martucci, AT '21]

- A quicker argument to show protection from D-brane bubbles: [Giri, Martucci, AT '21]

$$d_H(e^{2A-\phi}\Phi_1) = \frac{2}{L} e^{A-\phi}\mathrm{Re}\Phi_2 \qquad \Rightarrow \qquad d(e^{2A-\phi}e^{-B}\mathrm{Im}\Phi_1) = 0 \qquad H = dB$$

$$d_H(e^{3A-\phi}\mathrm{Im}\Phi_2) = \frac{3}{L} e^{2A-\phi}\mathrm{Im}\Phi_1 - e^{4A} * \lambda f$$

$$\Rightarrow \qquad \tfrac{3}{L}\left|\int_\Sigma e^{2A-\phi}e^{-\mathcal{F}}\mathrm{Im}\Phi_1\right| = \left|\int_\Sigma e^{4A}e^{-\mathcal{F}} * \lambda f\right| = |q|$$

$\mathcal{F} = B + 2\pi l_s^2 f$

Σ: wrapped by brane

$$/\!\!\wedge \qquad \text{calibration property}$$

$$\tfrac{3}{L}\int_\Sigma e^{2A-\phi}e^{-\mathcal{F}}\mathrm{vol} = \tau \qquad \Rightarrow \qquad \frac{|q|}{\tau} \leqslant 1$$

[equality: BPS]

- Can this protection be replicated for some non-susy vacua?

Stable susy breaking?

For this kind of stability, we don't need susy...

We really just need: $\mathrm{d}(\mathrm{e}^{2A-\phi}\mathrm{e}^{-B}\mathrm{Im}\Phi_1) = 0$

[so that $\int_\Sigma$ only depends on hom. class]

and

$$\frac{3}{L}\left|\int_\Sigma \mathrm{e}^{2A-\phi}\mathrm{e}^{-\mathcal{F}}\mathrm{Im}\Phi_1\right| \geqslant \left|\int_\Sigma \mathrm{e}^{4A}\mathrm{e}^{-\mathcal{F}} * \lambda f\right| \quad = |q|$$

$$\rotatebox{90}{$\geqslant$}$$

$$\frac{3}{L}\int_\Sigma \mathrm{e}^{2A-\phi}\mathrm{e}^{-\mathcal{F}}\mathrm{vol} \quad = \tau$$

$$\Rightarrow \quad \frac{|q|}{\tau} \leqslant 1$$

Can we find vacua with these properties?

There exist *fake* pure spinor equations:

[Lüst, Marchesano, Martucci, Tsimpis '08; Legramandi, AT '19, Menet '23]

- modification that breaks susy, still implies equations of motion
- generalization of GKP susy breaking [Giddings, Kachru, Polchinski '01]

However, so far this works best

- for Minkowski [Menet '23]
- keeping the $\partial_{\Phi_1} W = 0$ equation unchanged.

$$\mathrm{d}_H(\mathrm{e}^{3A-\phi}\mathrm{Im}\Phi_2) = -\mathrm{e}^{4A} * \lambda f$$

So we decided to look 'by hand'

[Menet, AT *WIP*]

Algebraic vacua

In several geometries, equations of motion become algebraic.

We found new classes, and analyzed stability for some known ones. [Menet, AT *WIP*]
Preliminary analysis

today

- $\mathrm{AdS}_4 \times$ Kähler–Einstein$_6$ in IIA — generalizations of old sol.
- $\mathrm{AdS}_4 \times$ homogeneous$_6$ in IIA — old & new
- $\mathrm{AdS}_5 \times$ (S^1-fibrations) in IIB — old & new

Bonus: similar methods also allow to find vacua in het. $\mathrm{SO}(16) \times \mathrm{SO}(16)$

[Raucci, AT *WIP*]

- AdS$_4\times$ Kähler–Einstein$_6$ in IIA
  with positive curvature

generalizes AdS$_4\times\mathbb{CP}^3$

[Gaiotto, AT '08]

$$F_{2k} = R^{2k} f_{2k} J^k$$

$$B = bJ$$

$$\Phi_1 = \mathrm{e}^{-\mathrm{i}J}$$

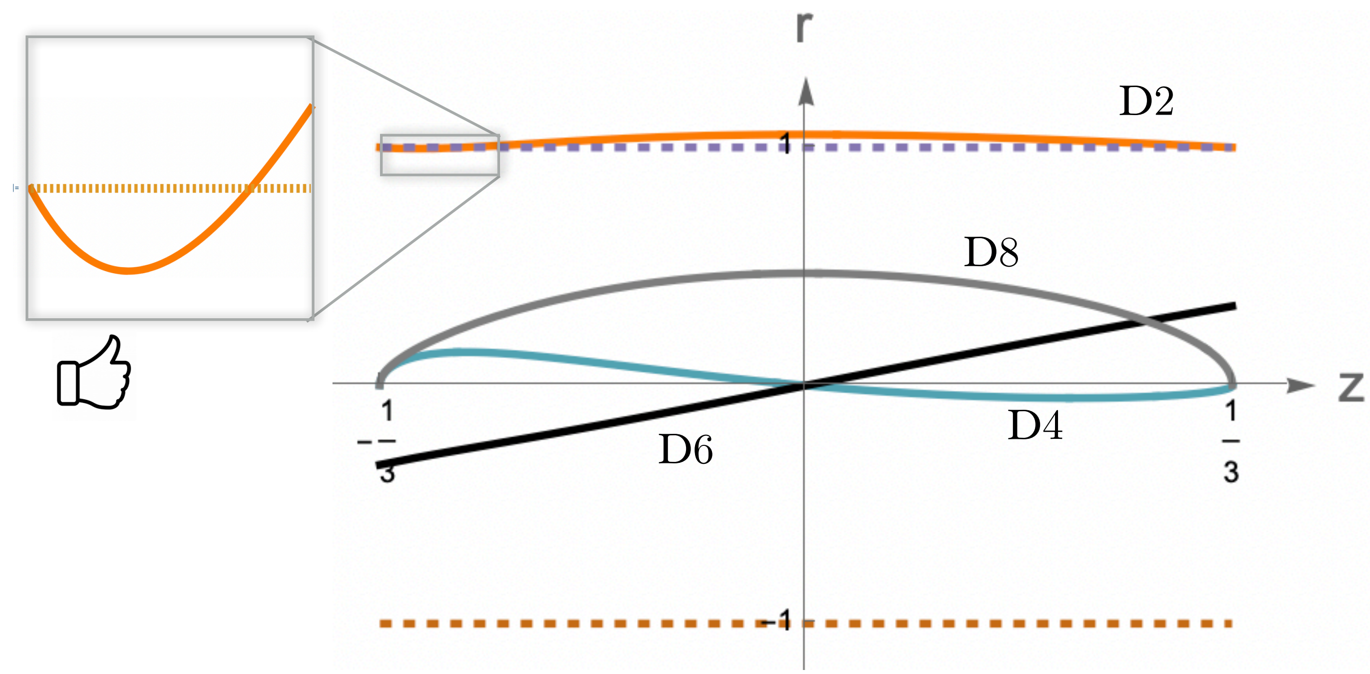
- analytic solution. Free: f_2, f_6, g_s, b
- flux quantization: each $\in \mathbb{Q}$

but

$$z \equiv \frac{f_2}{f_6} \in \left[-\tfrac{1}{3}, \tfrac{1}{3}\right]$$

$$|r_{\mathrm{D}p}| \equiv \left| \frac{\int_{\Sigma_{p-2}} \mathrm{e}^{4A} \mathrm{e}^{-\mathcal{F}} * \lambda f}{3/L \int_{\Sigma_{p-2}} \mathrm{e}^{2A-\phi} \mathrm{e}^{-\mathcal{F}} \mathrm{Im}\Phi_1} \right| \leqslant 1$$

in various degrees:



- D2 instability except for $z \in \left[-\tfrac{1}{3}, -2+\sqrt{3}\right]$
- more subtle discussion for bound states [Menet, AT WIP]

• for $\mathbb{CP}^3$, long ago:

noticing from (4.14) that $\left|\frac{f_6}{f_2}\right| \geq 3$, one finds (if f_6 and f_2 have equal sign) that $V \leq 0$: the electric repulsion term wins over the gravitational attractive term. These $\mathcal{N} = 0$ vacua are then non–perturbatively unstable towards nucleation of D2 branes. The dual field theories will then have an unstable potential.

[Gaiotto, AT '08]

[in our partial defense, we were mostly interested in the CFT dual]

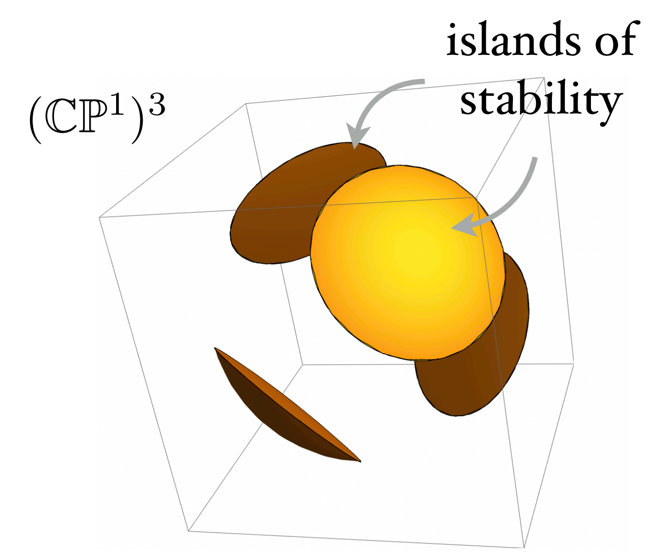
• $z = \pm 1/3$: $F_0 = 0$, uplift to 11d.

• $z = 1/3$: susy

• $z = -1/3$: skew-whiffed.

⇨ protection from tachyons near these endpoints.

• $\mathrm{KE}_6 = (\mathbb{CP}^1)^3$, $\mathrm{KE}_4 \times \mathbb{CP}^1$: new parameters



axes: $z_i \equiv \frac{f_{2i}}{f_6} \in \left[-\frac{1}{3}, \frac{1}{3}\right]$

- AdS$_4\times$ homogeneous$_6$ in IIA

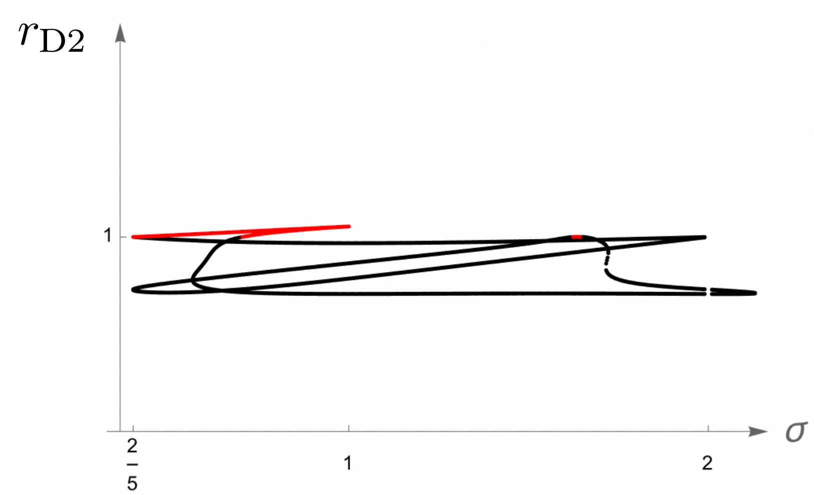
most interesting:

		metric parameters
•	$\frac{\mathrm{Sp}(2)}{\mathrm{Sp}(1)\times \mathrm{U}(1)} \cong \mathbb{CP}^3$	1
•	$\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times \mathrm{U}(1)} \cong \mathbb{F}(1,2;3)$	2

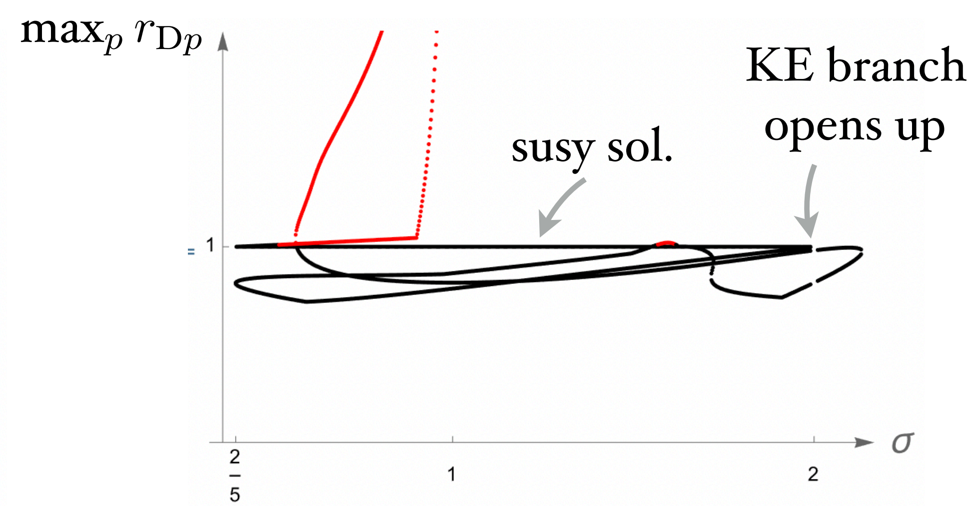
- *twistor* fibrations $S^2 \hookrightarrow M_6 \to M_4 = S^4, \mathbb{CP}^2$
- also admit susy vacua [AT '07, Koerber, Lüst, Tsimpis '08]
- $\mathbb{CP}^3$: solutions already found. [Koerber, Körs '10]

just for illustration purposes:

we find that most solutions are stable:



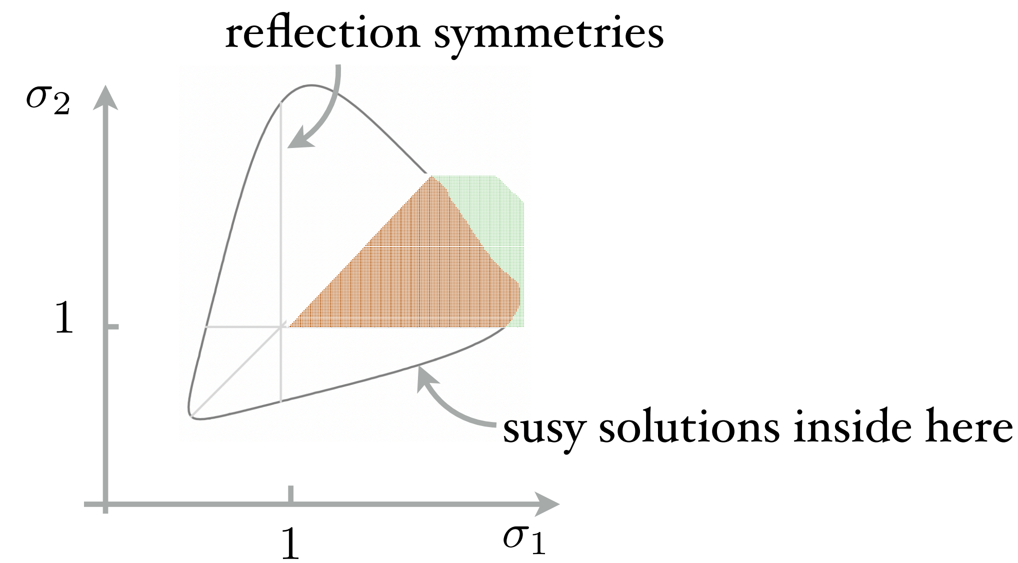
red = instability
black = stability



red = we cannot conclude instability
black = stability

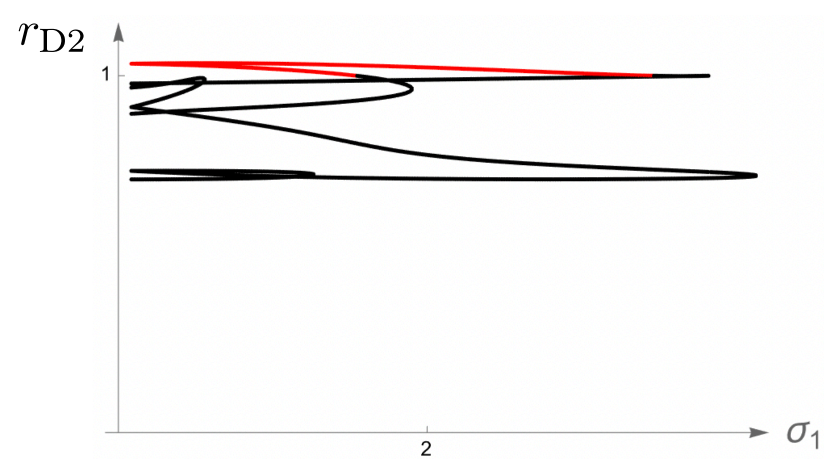
• $\mathbb{F}(1,2;3)$: two metric parameters

new solutions:



red= some solutions can be unstable
green= all solutions are stable

a slice at $\sigma_2 = 3/2$:



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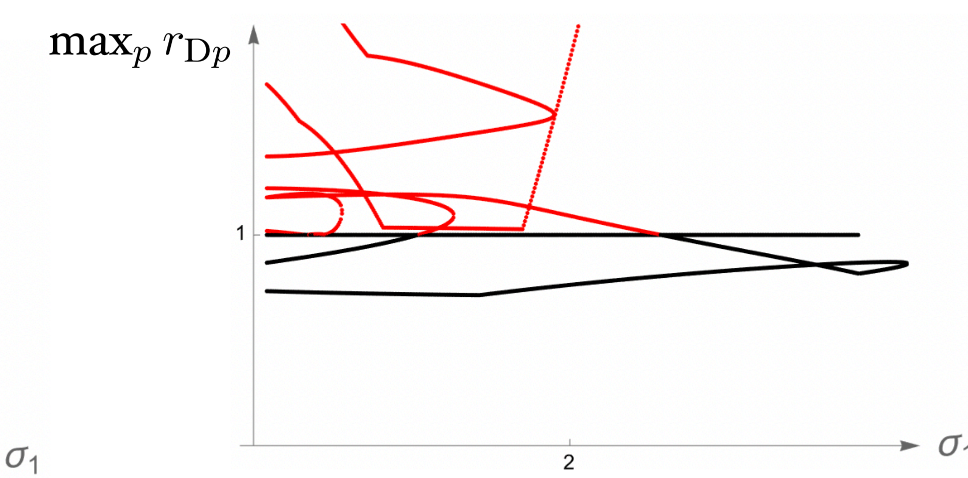

red = we cannot conclude instability
black = stability

Summary: many solutions are stable against brane bubbles. [Menet, AT WIP]

It's *much* harder to protect against more general vacuum decay!

- Positivity theorems with fake susy?

  didn't quite work in M-theory, much more open in type II. [Giri, Martucci, AT '21]

- *Bubble of nothing* is a worrisome prospect whenever there are circles...

  or more generally spheres: potentially relevant for $\mathbb{CP}^3$ [Ooguri, Spodyneiko '17]

  A generic KE appears safe from this point of view.

Conclusions

- Finding all non-susy vacua might not be a very fruitful endeavor;
  focusing on potentially/partially stable ones might be sensible

- Pure spinors and G-structures give a natural way to do so

- Relatively easy to find solutions that are stable under brane bubbles

- It will be a lot harder to provide full stability!
  We might fail, but learn something useful in the process

Backup Slides

Fake susy

[Boucher '84;... Freedman, Nunez, Schnabl, Skenderis '03; Lüst, Marchesano, Martucci, Tsimpis '08; Legramandi, AT '19, Menet '23]

A modified pure spinor equation that still implies equations of motion:

$$\left.\begin{aligned} &\mathrm{d}_H(\mathrm{e}^{2A-\phi}\Phi_1)=0 \\ &\cancel{\mathrm{d}_H(\mathrm{e}^{A-\phi}\mathrm{Re}\Phi_2)=0} \\ &\mathrm{d}_H(\mathrm{e}^{3A-\phi}\mathrm{Im}\Phi_2)=\mathrm{e}^{4A}*\lambda f \end{aligned}\right\} \Rightarrow$$

equations of motion can be written in terms of

$$\Xi\equiv\mathcal{P}\mathrm{d}_H(\mathrm{e}^{A-\phi}\mathrm{Re}\Phi_2)$$

[Menet '23]

$$\mathcal{P}\equiv *-\frac{4}{\mathrm{vol}}\left((\gamma^m\mathrm{Re}\Phi_2,\,\cdot\,)_6\gamma_m\mathrm{Re}\Phi_2+(\mathrm{Re}\Phi_2\gamma^m,\,\cdot\,)_6\mathrm{Re}\Phi_2\gamma_m\right)$$

For example, dilaton EoM: $(\mathrm{d}_H(\mathrm{e}^{A-\phi}\mathrm{Im}\Phi_2),\Xi)_6=0$

internal Einstein eq: $(g^{mn}\mathrm{d}_H(\mathrm{e}^{A-\phi}\mathrm{Im}\Phi_2)-[\mathrm{d}y^{(m}\wedge\iota^{n)},\mathrm{d}_H]\mathrm{e}^{A-\phi}\mathrm{Im}\Phi_2,\Xi)_6=0$

- The same strategy might become more effective in terms of exceptional geometry [Menet, Petrini, Waldram *WIP*]
- Implications on stability? [Boucher '84; Giri, Martucci, AT '21]

Details of KE solution

$$F_{2k} = R^{2k} f_{2k} J^k$$

$$B = bJ$$

- analytic solution. Free: f_2, f_6, g_s, b

$$f_0^2 = \frac{(1-9z^2)(1+2z)^2}{5+20z+23z^2} f_6^2 \qquad R = 2L_{\text{AdS}}$$

$$f_4 = -\frac{f_0 f_2}{f_6+2f_2} \qquad \frac{32}{g_s^2 R^2} = \frac{4(1+z)^4}{5+20z+23z^2} f_6^2$$

- flux quantization: each $\in \mathbb{Q}$

$$\frac{(n_2^2-2n_0n_4)^3}{(n_2^3+3n_0^2n_6-3n_0n_2n_4)^3} =$$