The Extreme D3-brane

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Work in progress with Jorge Russo
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(2211.10689, 2311.04278)

April 4, 2025

Motivations

Born-Infeld is unique theory of nonlinear electrodynamics (NLED) with a weak-field limit for which shock waves propagate *without birefringence*.
The only causal no-birefringence NLED theories are

- Born-Infeld (BI) [Boillat, Plebanski, c. 1970]
- Bialynicki-Birula (BB) [Bialynicki-Birula, 1983, 1992]
- Extreme BI (EBI) [Russo, PKT '22. Mezincescu, Russo, PKT '23]

These are *BI and its limits*

BI describes the electromagnetic interactions on the worldvolume of a static planar D3-brane of IIB superstring theory [Fradkin & Tseytlin 1985, Bergshoeff, Pope, Sezgin, PKT, 1987, Leigh 1989, Tseytlin 1999, ...].

➽ $\boxed{\text{Is there an extreme limit of the D3-brane?}}$

D3-brane in a IIB Minkowski vacuum

10D Metric is Minkowski: $g = \eta$. Vacuum value of dilaton-field determines IIB string coupling constant g_s. For simplicity choose $\boxed{\textit{zero axion field}}$.
Let T_1 be IIB F(undamental)-string tension.

Bosonic truncation of effective field theory for D3-brane is the 4D Dirac-Born-Infeld (DBI) theory with

$$\mathcal{L}_{DBI} = -T_3\sqrt{-\det\left(G + F/T_1\right)}\,, \qquad T_3 = \frac{T_1^2}{g_s}$$

G is induced 4D worldvolume metric and $F = dA$ a 2-form field-strength for independent worldvolume 1-form potential A.

➽ Recall that D-string tension is

$$\tilde{T}_1 = T_1/g_s \qquad \left(\Leftrightarrow \quad T_1 = \tilde{T}_1/\tilde{g}_s\,, \quad \tilde{g}_s = 1/g_s\right)$$

Alternative formulations

In terms of the (pseudo)scalar densities

$$S = -\frac{1}{4}\sqrt{-\det G}\, G^{\mu\rho} G^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma}\,, \qquad P = -\frac{1}{8}\varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

we have

$$\mathcal{L}_{DBI} = -\frac{1}{g_s}\sqrt{\mathcal{T}^2 - 2\mathcal{T}S - P^2}\,, \qquad \mathcal{T} := T_1^2\sqrt{-\det G}\,.$$

➳ Equivalently [following Roček-Tseytlin for BI] we can linearise in (S, P) by introducing auxiliary scalar fields (u, v):

$$\mathcal{L}_{DBI}^{(RT)} = -\frac{\mathcal{T}}{2}\left\{ v + \frac{(1 + g_s^2 u^2)}{g_s^2 v} \right\} + vS + uP\,.$$

Diff-invariant phase-space formulation

This is a manifestly 10D Lorentz invariant formulation with Hamiltonian constraints generating 4D worldvolume diffeomorphisms.

For Minkowski coordinates $\{X^m; m = 0, 1, \ldots, 9\}$ and worldvolume coordinates $x^\mu = (t, \sigma^i)$,

$$\mathcal{L} = \dot{X}^m P_m + D^i E_i - s^i \left\{\partial_i X^m P_m - \varepsilon_{ijk} D^j B^k\right\}$$
$$-\tfrac{1}{2}\ell \left\{\eta^{mn} P_m P_n + T_1^2 D^i D^j h_{ij} + \tilde{T}_1^2 B^i B^j h_{ij} + (T_1 \tilde{T}_1)^2 \det h\right\}$$

Elimination of conjugate momentum variables (P_m, D^i) yields Lagrangian with (s^i, ℓ) as auxiliary fields; eliminating them yields $\mathcal{L}_{DBI}$.

➽ Note manifest $U(1)$ duality invariance, which acts by phase shift of complex 3-vector field $\mathbf{D} + (i/g_s)\mathbf{B}$.

Monge Gauge

Fix worldvolume diffeos by Monge gauge choice

$$X^m = \left\{t, \boldsymbol{\sigma}, \vec{X}(t,\boldsymbol{\sigma})\right\}, \qquad P_m = \left\{-\mathcal{H}, \mathbf{P}, \vec{\Pi}(t,\boldsymbol{\sigma})\right\}.$$

Now we can solve constraints for $(\mathcal{H}, \mathbf{P})$. For example, for a planar static 3-brane ($\boldsymbol{\nabla}\vec{X} = \vec{\mathbf{0}}$ and $\vec{\Pi} = \vec{0}$) we have $\mathbf{P} = \mathbf{D} \times \mathbf{B}$ and

$$\mathcal{H} = \sqrt{T_3^2 + 2sT_3 + p^2}\,,$$

where

$$2s = g_s|\mathbf{D}|^2 + \frac{1}{g_s}|\mathbf{B}|^2\,, \qquad p = |\mathbf{D} \times \mathbf{B}|$$

This is the Born-Infeld Hamiltonian density (generalised to arbitrary g_s).

Electric EDBI

Electric Extreme limit is a strong-coupling limit:

$$\boxed{g_s \to \infty \quad \text{for fixed } \; T_1} \qquad (\tilde{T}_1 \to 0 \quad \& \quad T_3 \to 0)$$

but $2sT_3 \to T_1^2|\mathbf{D}|^2$, and therefore (again for planar static D3-brane)

$$\mathcal{H} \to \mathcal{H}_{eBI} = \sqrt{T_1^2|\mathbf{D}|^2 + |\mathbf{D} \times \mathbf{B}|^2}\,.$$

This defines 'electric' EBI. Get Lagrangian either by Legendre transform or from extreme limit of $\mathcal{L}_{BI}^{(RT)}$. After elimination of u we have

$$\mathcal{L}_{eEBI} = \lambda \left(T^2 - 2TS - P^2\right) \qquad (2\lambda = -v/T)$$

➻ This constraint implies that $|\mathbf{E}|$ takes its "critical" value.
(cf. Seiberg, Susskind Toumbas, 2000)

Magnetic DEBI

Magnetic Extreme limit: $\boxed{\tilde{g}_s \to \infty \quad \text{for fixed} \quad \tilde{T}_1}$

This just reverses roles of **D** and **B**. We now get (planar static D3-brane)

$$\mathcal{H} = \sqrt{\tilde{T}_1^2 |\mathbf{B}|^2 + |\mathbf{D} \times \mathbf{B}|^2}\,.$$

This defines 'magnetic' EBI. To get Lagrangian, either take Legendre transform or take 'magnetic' extreme limit of $\mathcal{L}_{BI}^{(RT)}$. Either way, one gets

$$\mathcal{L}_{mEBI} = -T_1\sqrt{-2S} \; + \; uP \qquad (|\mathbf{B}|^2 > |\mathbf{E}|^2)$$

Now the constraint is $P = 0$. **N.B.** $\boxed{\partial_i D^i = \partial_j B^j = 0}$.

➽ Looks different from 'electric' case, but the physics must be equivalent!

Susy preservation

IIB Minkowski vacuum has 32 Killing spinors: $\{\epsilon\}$. Number of susys preserved by D3-brane is number of solutions to equation of form

$$\Gamma(t,\boldsymbol{\sigma})\epsilon = \epsilon\,, \qquad \Gamma^2 \equiv \mathbb{I}_{32}$$

Focus on static magnetic DBI with $|\mathbf{E}| = 0$ ($\Leftrightarrow$ $|\mathbf{D}\times\mathbf{B}| = 0$) and choose $X^m = \{t, X^a; a = 1,\dots,9\}$. Then, since $\boxed{T_1 = 0\,,\ g_s = 0}$ we get

$$\mathcal{H} = \tilde{T}_1\sqrt{h_{ij}B^iB^j}\,, \qquad h_{ij} = \partial_i X^a\cdot\partial_j X^b\delta_{ab}\,.$$

Susy condition is $\boxed{(\tau_1\otimes\gamma_0\Gamma_a)\left(B^i\partial_i X^a\right)\epsilon = \sqrt{h_{ij}B^iB^j}\,\epsilon}$

➽ $\mathbf{B} = (B,0,0)$ and $X^a = (\sigma^1,\mathbb{X})$, with $\partial_1 B = \partial_1\mathbb{X} = 0 \rightarrow$ 16 susys.

Where are the strings?

The 16-susy magnetic EDBI-brane is a string with tension

$$\frac{d\mathcal{H}}{d\sigma^1} = \tilde{T}_1 \int d\sigma^2 d\sigma^3 B(\sigma^2, \sigma^3) = \tilde{T}_1 \langle B \rangle \mathcal{A}$$

where $\langle B \rangle$ is average B. Cf. Landau problem: plane orthogonal to $\mathbf{B}$ is non-commutative, and minimum area is $1/B$. For $\mathcal{A} = 1/\langle B \rangle$ we get a string of tension $\tilde{T}_1$. This is the D-string dissolved into an EDBI brane.

➸ Where are the tensionless (and non-interacting) F-strings?

These are now tensionless flux tubes of $\mathbf{D} = (D, 0, 0)$ (with $\partial_1 D = 0$) dissolved in the tensionless 3-brane. End points are $+$ and $-$ electric charges. They move inertially in the direction of $\mathbf{B}$.

Outlook

Many questions remain.

- What is connection to NCOS?
- Are there any 1/4 BPS solutions
- Questions I did not think of yet but which you may be going to ask