Emergent Strings: UV and IR

with Lukas Kaufmann and Stefano Lanza, arXiv 2412.12251

with Björn Friedrich, Jeroen Monnee and Max Wiesner, arXiv: 2504.01066

Timo Weigand, Gravity, Strings and Supersymmetry Breaking, 05/04/2025

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Motivation

Long-term question in fundamental theoretical physics:

Which properties must a *consistent* quantum gravity theory in $d \geq 4$ have?

GRAVITY IS DIFFERENT FROM

QUANTUM FIELD THEORY

Which properties must a consistent QG have?

Example: Characteristic behaviour of QG near asymptotic boundaries of field space

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parametrically weak coupling

Distance Conjecture **[Ooguri,Vafa'06]**

At infinite distance in moduli space of QG, a tower of states becomes light exponentially fast.

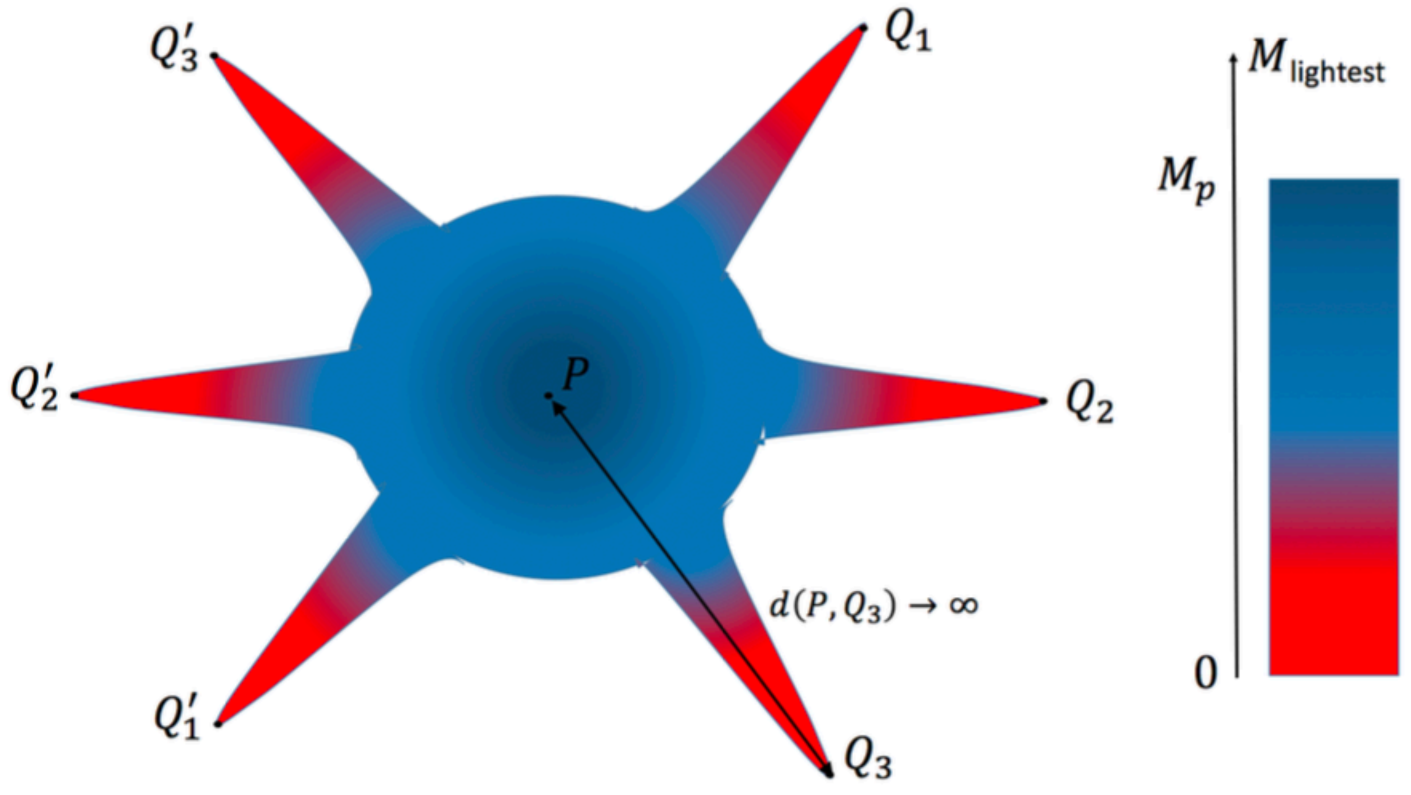


Image: Palti,1903.06239

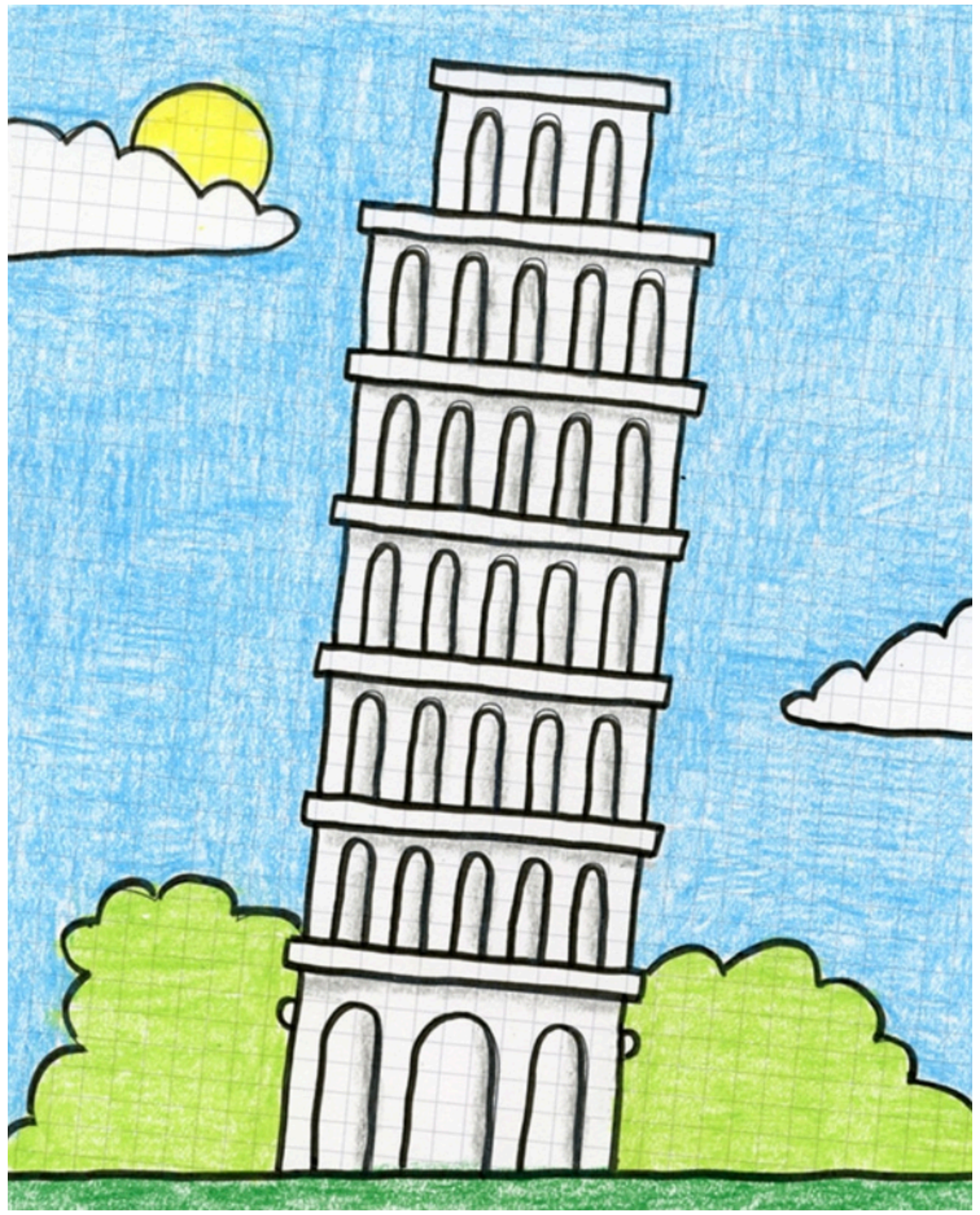
Emergent String Conjecture **[Lee,Lerche,TW'19]**

At infinite distance, all theories either decompactify or become a weakly coupled string theory.

Emergent String Conjecture

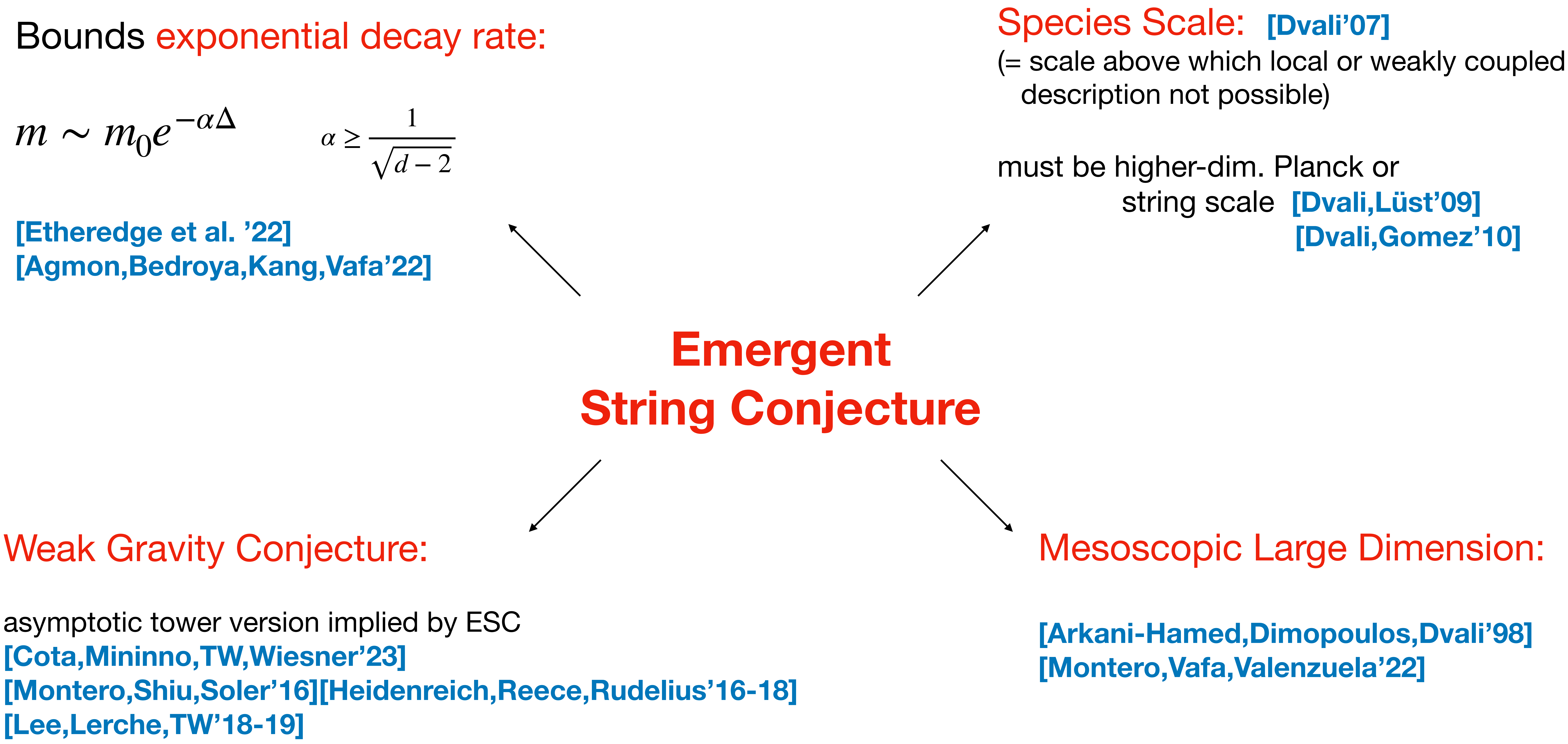
Two types of light weakly coupled towers:

Kaluza-Klein tower



Excitation tower of unique critical string

accompanied by KK tower if $d < 10$



Emergent String Conjecture

Bounds exponential decay rate:

$$m \sim m_0 e^{-\alpha\Delta} \qquad \alpha \geq \frac{1}{\sqrt{d-2}}$$

[Etheredge et al. '22]
[Agmon,Bedroya,Kang,Vafa'22]

Species Scale: **[Dvali'07]**

(= scale above which local or weakly coupled description not possible)

must be higher-dim. Planck or string scale **[Dvali,Lüst'09]** **[Dvali,Gomez'10]**

Weak Gravity Conjecture:

asymptotic tower version implied by ESC
[Cota,Mininno,TW,Wiesner'23]
[Montero,Shiu,Soler'16][Heidenreich,Reece,Rudelius'16-18]
[Lee,Lerche,TW'18-19]

Mesoscopic Large Dimension:

[Arkani-Hamed,Dimopoulos,Dvali'98]
[Montero,Vafa,Valenzuela'22]

Evidence

Microscopic /
ultra-violet input

Effective field theory /
infrared input

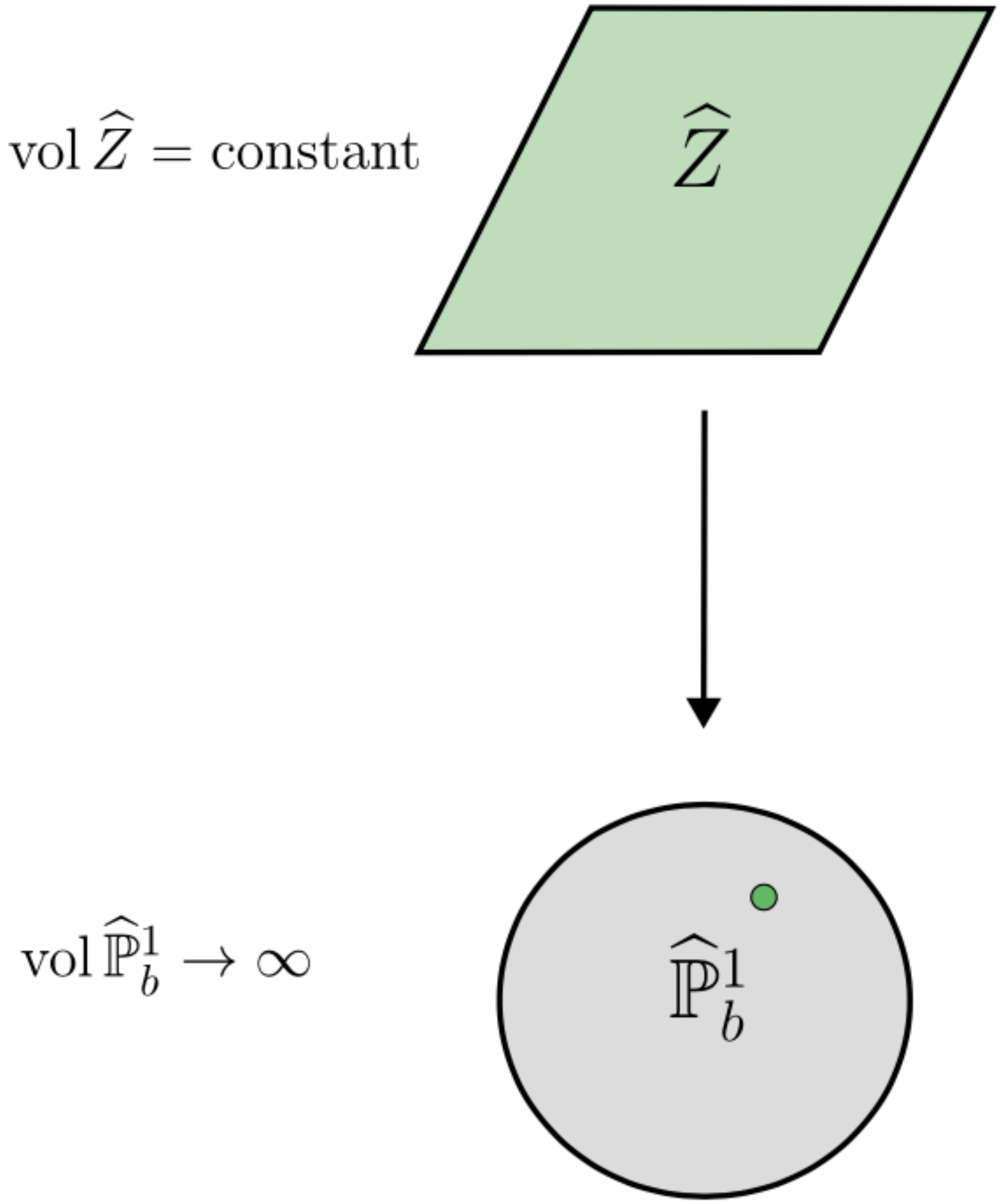
Microscopic / ultra-violet evidence

Compactifications of string or M-theory:

Ingredients:

1) Higher-dimensional branes

2) Degenerating Calabi-Yau geometry



Microscopic / ultra-violet evidence

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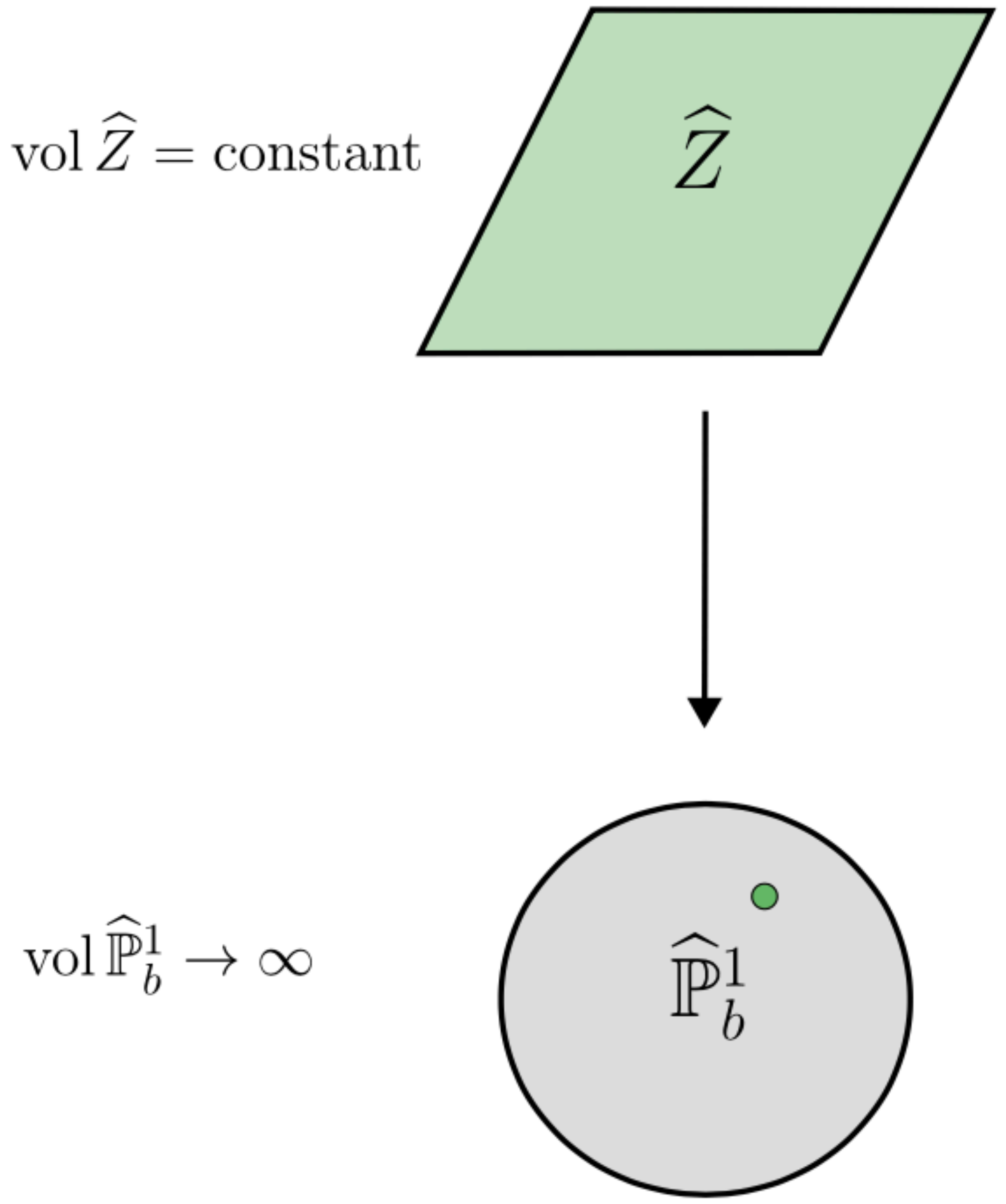


- Kähler sector M/F-theory in 6d/5d/4d [Lee,Lerche,TW'18-20]
- Complex structure F-theory 6d/8d [Lee,Lerche,TW'21] [Alvarez-Garcia,Lee,TW'23]
- Type II / M-theory hypermultiplets [(Baume),Marchesano,Wiesner'20] [Alvarez-Garcia,Kläwer,TW'22] [Blumenhagen,Gligovic,Paraskevopoulo'23]
- Non-supersymmetric strings [Basile'22]
- Non-geometric strings [Aoufia,Basile,Leone'24]

……

$\Longrightarrow$ Demonstrated ESC explicitly, but relies heavily on UV input

Effective Field Theory / Infra-red evidence

1) **Asymptotic behaviour of couplings**, central charges, masses, kinetic terms

- Type IIB asymptotic Hodge theory

  [Grimm,Palti,Valenzuela'18] combined with

  EFT string solutions of 4d N=2 SUGRA

  [Lanza,Marchesano,Martucci,Valenzuela'21]

$\Longrightarrow$

Candidate for emergent string, but remains to prove:

1) Criticality and uniqueness

2) Existence of accompanying particle tower

2) **Bottom-up arguments**

- Black hole/Species entropy
  [Basile,(Cribiori),Lüst,Montella'23/24]
  [Herraez,Lüst,Masias,Scalisi'24]

$\Longrightarrow$

Constrains towers to be Kaluza-Klein like

or

consistent with tower of string excitations

- Scattering amplitudes
  [Bedroya,Mishra,Wiesner'24]

New UV and IR techniques to understand ESC

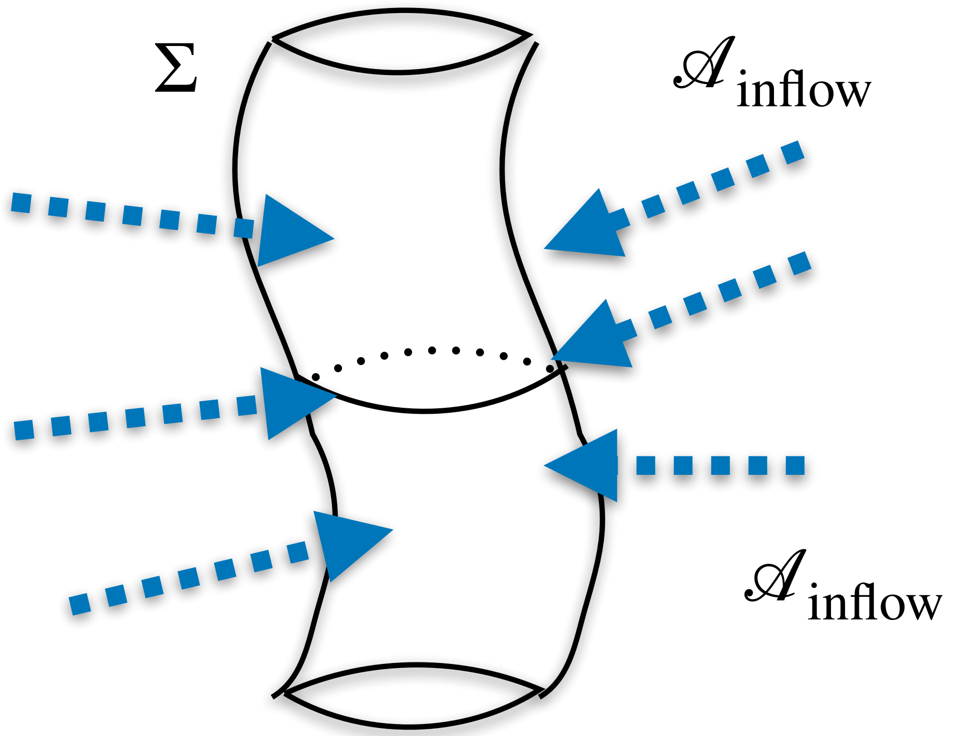
i) IR perspective

5d N=1 supergravity bottom - up:

What replaces input from geometry?

⇕

New constraints from probe branes



[Kaufmann,Lanza,T.W.’24]

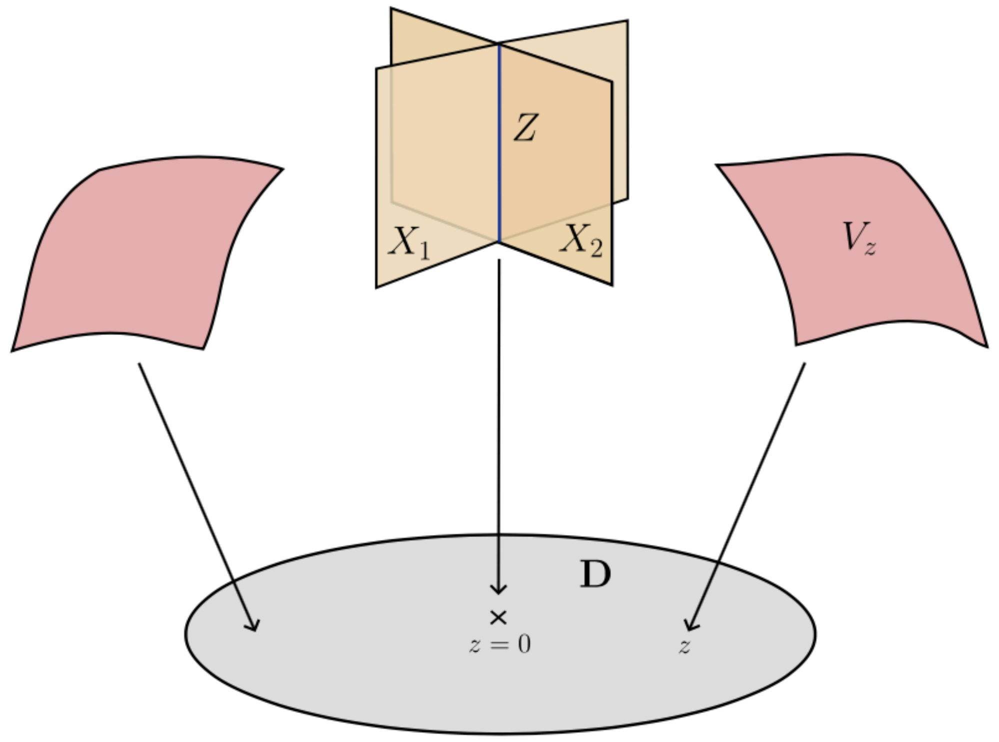
ii) UV perspective

4d N=2 Type IIB on CY3 with complex structure deg.:

What replaces higher-dimensional brane?

⇕

Geometry of complex structure degenerations



[Friedrich,Monnee,T.W.,Wiesner’25]

ESC in 5d N=1 supergravity –

via probe arguments

Kaufmann, Lanza, TW, arXiv: 2412.12251

5d N=1 Supergravity

$$S = \int_{\mathbb{R}^{1,4}} \left(\frac{M_{\mathrm{P}}^3}{2} \star R - \frac{M_{\mathrm{P}}}{4} f_{IJ} F^I \wedge \star F^J - \frac{1}{12} \mathscr{F}_{IJK} A^I \wedge F^J \wedge F^K + \text{scalars} + \text{hypers} \right)$$

@ 2-derivative level

focus on
vector fields/mutliplets: $A^I, \quad I = 0, i$

X^I: Inhomogenenous coordinates on **vector multiplet moduli space** subject to $\mathscr{F}[X] = 1$

Prepotential: $\mathscr{F}[X] = \frac{1}{3!} \mathscr{F}_{IJK} X^I X^J X^K$

Gauge kinetic matrix: $f_{IJ} = \mathscr{F}_I \mathscr{F}_J - \mathscr{F}_{IJ}$

$$\mathscr{F}_I := \frac{\partial \mathscr{F}}{\partial X^I} = \frac{1}{2} \mathscr{F}_{IJK} X^J X^K, \qquad \mathscr{F}_{IJ} := \frac{\partial^2 \mathscr{F}}{\partial X^I \partial X^J} = \mathscr{F}_{IJK} X^K$$

BPS particles and strings

BPS Particles of charge q_I:

$$S_{\text{part}} = -\int_{\mathscr{L}} \mathrm{d}\tau\sqrt{-\gamma}\, M_{\text{part}} + q_I \int_{\mathscr{L}} A^I$$

$$M_{\text{part}} = M_{\text{P}}\, q_I X^I$$

BPS Strings of charge p^I:

$$S_{\text{str}} = -\int_{\mathscr{W}} \mathrm{d}^2\xi\sqrt{-h}\, T_{\text{str}} + p^I \int_{\mathscr{W}} B_I$$

$$T_{\text{str}} = \frac{M_{\text{P}}^2}{2} p^I \mathscr{F}_I$$

$$\mathscr{F}_I := \frac{\partial \mathscr{F}}{\partial X^I}$$

Supergravity strings: charges in cone dual to BPS particle cone

do not decouple from gravity

[Katz,Kim,Tarazi,Vafa'20]

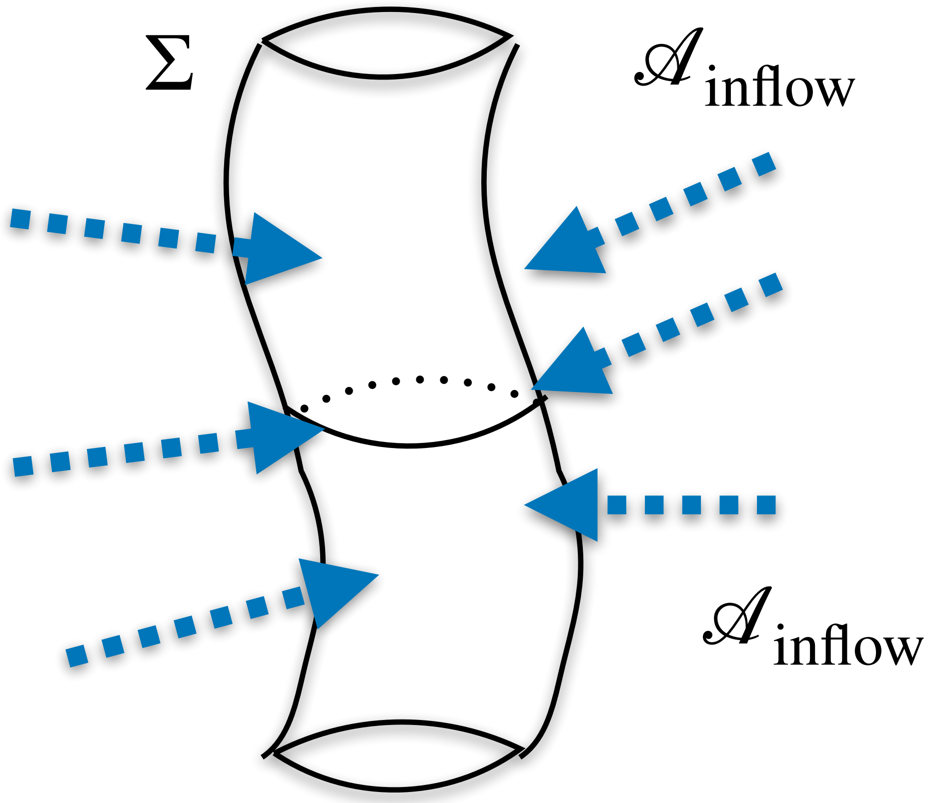
Constraints from SUGRA strings

Anomaly inflow from bulk ⟹ anomaly on worldsheet of strings

[Ferrara,Khouri,Minasian'96]

....

$$I_4|_{\text{bulk}} = -\frac{1}{2}k^{(\text{p})}_{JK}A^J \wedge A^k \quad = \quad I_4|_{\text{string},1-\text{loop}}$$



't Hooft anomaly matrix: $k^{(\text{p})}_{JK} = p^I \mathscr{F}_{IJK}$

Supergravity strings: $\text{sgn}(k^{(\text{p})}) = (1,r)\,, \qquad r \leq n_V - 1$

[Katz,Kim,Tarazi,Vafa'20]

right moving
universal scalar

left-moving currents/
fermions induced by bulk fields

Constraints from SUGRA strings

Supergravity strings: $\mathrm{sgn}(k^{(\mathrm{p})}) = (1,r)\,, \qquad r \leq n_V - 1$

$$k^{(p)}_{IJ} \simeq \begin{pmatrix} +1 & 0 & 0 & \dots & 0 \\ 0 & -1 & 0 & \dots & 0 \\ 0 & 0 & -1 & \dots & 0 \\ 0 & 0 & 0 & \dots & -1 \end{pmatrix}$$

Consequences: Given a Kähler basis (X^I in cone dual to BPS q_I)

i) **Positivity**: $\mathscr{F}_{IJK} \geq 0 \quad \forall I, J, K$

Relations known to hold if Sugra comes from CY3 compactificaton

ii) Various other **non-trivial relations**, for example:

$$\mathscr{F}_{000} = 0\,, \quad \mathscr{F}_{00i} \neq 0 \quad \mathscr{F}_{00r} = 0 \quad \Longrightarrow \mathscr{F}_{0ir} \neq 0$$

$$c_i \mathscr{F}_{0jr} = c_j \mathscr{F}_{0ir} \qquad \forall r$$

[Kaufmann,Lanza,TW'24]

Infinite Distance Limits

Consider a patch in physical region/Kähler cone of vector moduli space parametrised by $X^I \geq 0$

Infinite distance limit:

- For some X^0 : $X^0 \sim \lambda \to \infty$
- For all remaining X^a : $X^a \lesssim \lambda$
- Keep hypersurface constraint: $1 = \mathscr{F}_{IJK} X^I X^J X^K$

a priori
divergent or not

$$1 = \mathscr{F}_{IJK} X^I X^J X^K \quad \Longrightarrow \quad \mathscr{F}_{000} \overset{!}{=} 0$$

Uses $\mathscr{F}_{IJK} \geq 0$

Class A:

$\mathscr{F}_{00i} \neq 0$ for some $i \in \mathscr{I}_1$

Class B:

$\mathscr{F}_{00I} = 0$ for all I

Infinite Distance Limits

Infinite distance limit:

- For some X^0 : $X^0 \sim \lambda \to \infty$
- For all remaining X^a : $X^a \lesssim \lambda$
- Keep hypersurface constraint: $1 = \mathscr{F}_{IJK} X^I X^J X^K$

$\implies \mathscr{F}_{000} = 0$

Class A:

$\mathscr{F}_{00i} \neq 0$ for some $i \in \mathscr{I}_1$

Class B:

$\mathscr{F}_{00I} = 0$ for all I

$\implies$ Previous relations for CS terms become relevant -

such as $\mathscr{F}_{000} = 0\,,\quad \mathscr{F}_{00i} \neq 0 \quad \mathscr{F}_{00r} = 0 \quad \implies \mathscr{F}_{0ir} \neq 0$

Result of classification

[Kaufmann,Lanza,TW'24]

Using consistency of SUGRA strings: Every vector multiplet infinite distance limit is

- either a **vector limit:**

A **1-form** becomes **weakly coupled** faster than any tensor field (coupling to a SUGRA string)

$\mathbf{q}^2_{\min} \prec \mathcal{Q}(\mathbf{p}) \quad \forall \mathbf{p}$ of SUGRA strings

- or a **tensor limit:**

Precisely one tensor field becomes **weakly coupled** faster than any other tensor

and

always accompanied by at least one **1-form** that becomes **weakly coupled at same rate**

$\mathcal{Q}_{\min}(\mathbf{p}_0) \sim \mathbf{q}^2_{\min}$

Result of classification

[Kaufmann,Lanza,TW'24]

Using consistency of SUGRA strings: Every vector multiplet infinite distance limit is

- either a vector limit:

$$\mathbf{q}_{\text{min}}^2 \prec \mathscr{Q}(\mathbf{p}) \quad \forall \mathbf{p} \text{ of SUGRA strings}$$

for precisely one leading $U(1)$

$$\mathbf{q}_{\text{min}} \sim \exp\left(-\frac{2}{\sqrt{3}}\ell(\lambda)\right) \qquad \ell(\Lambda) = \int_0^\lambda \sqrt{\frac{1}{2} f_{IJ} \dot{X}^I \dot{X}^J} d\lambda'$$

Consistent with:
circle decompactification from d=5 to d=6

- or a tensor limit:

$$\mathscr{Q}_{\text{min}}(\mathbf{p}_0) \sim \mathbf{q}_{\text{min}}^2$$

$$\mathbf{q} \sim \exp\left(-\sqrt{\frac{1}{d-2}}\bigg|_{d=5} \ell(\lambda)\right)$$

Consistent with:
weak coupling limit for a critical string

Consistent with Emergent String Conjecture

cf. [Etheredge,Heidenreich,Kaya,Qiu,Rudelius'22] [Agmon,Bedroya,Kang,Vafa'22]

Open: Criticality?

[Kaufmann,Lanza,TW'24]

Remaining question from bottom-up:

Prove that string coupling to weakly coupled 2-form is a critical string

Necessary condition: $c_L = 24, \quad c_R = 12$ (heterotic) or $c_L = c_R = 12$ (Type II)

$c_L - c_R$ encoded in $C_0 = \frac{1}{2}(c_L - c_R)$ $\qquad S_{ARR} = \frac{1}{192} \int_{\mathbb{R}^{1,4}} C_I A^I \wedge \mathrm{tr}\left(\mathscr{R} \wedge \mathscr{R}\right)$

Conjecture: $C_{00I} = 0 \quad \forall I \qquad \Longrightarrow \qquad C_0 \in \{0,24\}$

How show this via probe string arguments?

ESC in Type IIB complex structure limits

Friedrich, Monnee, TW, Wiesner, arXiv: 2504.01066

ESC in Type IIB complex structure limits

CY 3 complex structure degenerations classified by asymptotic Hodge theory algebraically:

Type II, Type III, Type IV at infinite distance of CY 3-fold [Grimm,Palti,Valenzuela'18], …

1) Certain **3-cycle volumes vanish**:

$$\mathrm{vol}(\Gamma)\, M_{\mathrm{Pl}}^3 \to 0$$

$$\Downarrow$$

D3-branes on cycle Γ gives **massless particles**

$$\frac{M_\Gamma^2}{M_{\mathrm{Pl}}^2} \to 0$$

2) Certain **axions become weakly coupled** in **4d N=2 SUGRA**

$$\Downarrow$$

[Lanza,Marchesano,Martucci, Valenzuela'21]

Magnetic string becomes **tensionless:**

$$\frac{T_{\mathrm{str}}}{M_{\mathrm{Pl}}^2} \to 0$$

ESC in Type IIB complex structure limits

Type II degenerations: $M_\Gamma^2 \sim T_{\text{str}} \to 0$ for lightest such states

Candidate for accompanying KK tower if a Γ can be multi-wrapped

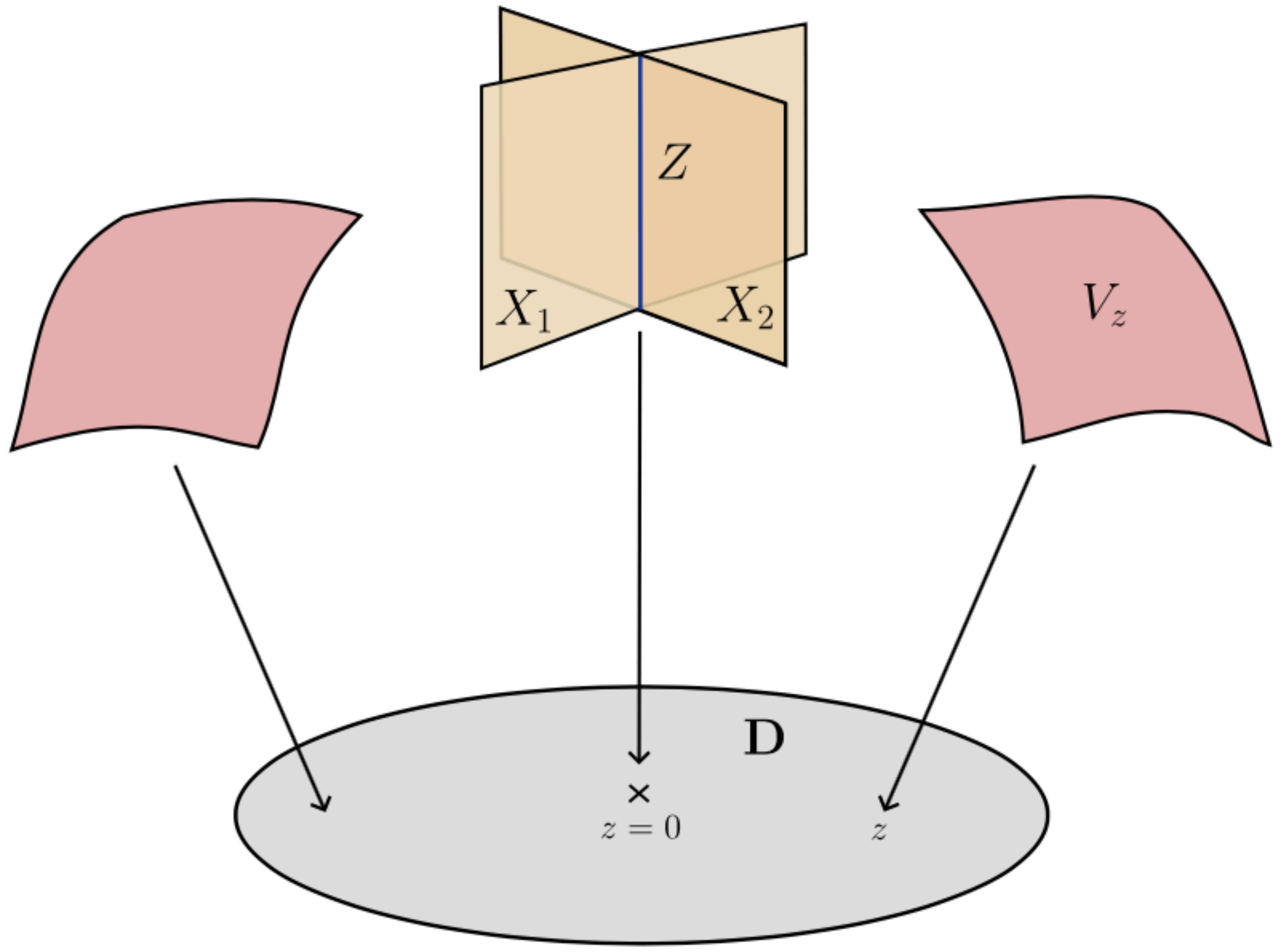
Candidate for emergent string - if it is a critical string

Required: information beyond algebraic classification of Hodge theory:

Geometric input needed

Address this in a priori special class of Type II degenerations:

Tyurin degenerations $\mathrm{CY}_3 = X_1 \cup_Z X_2 \quad Z = K3$



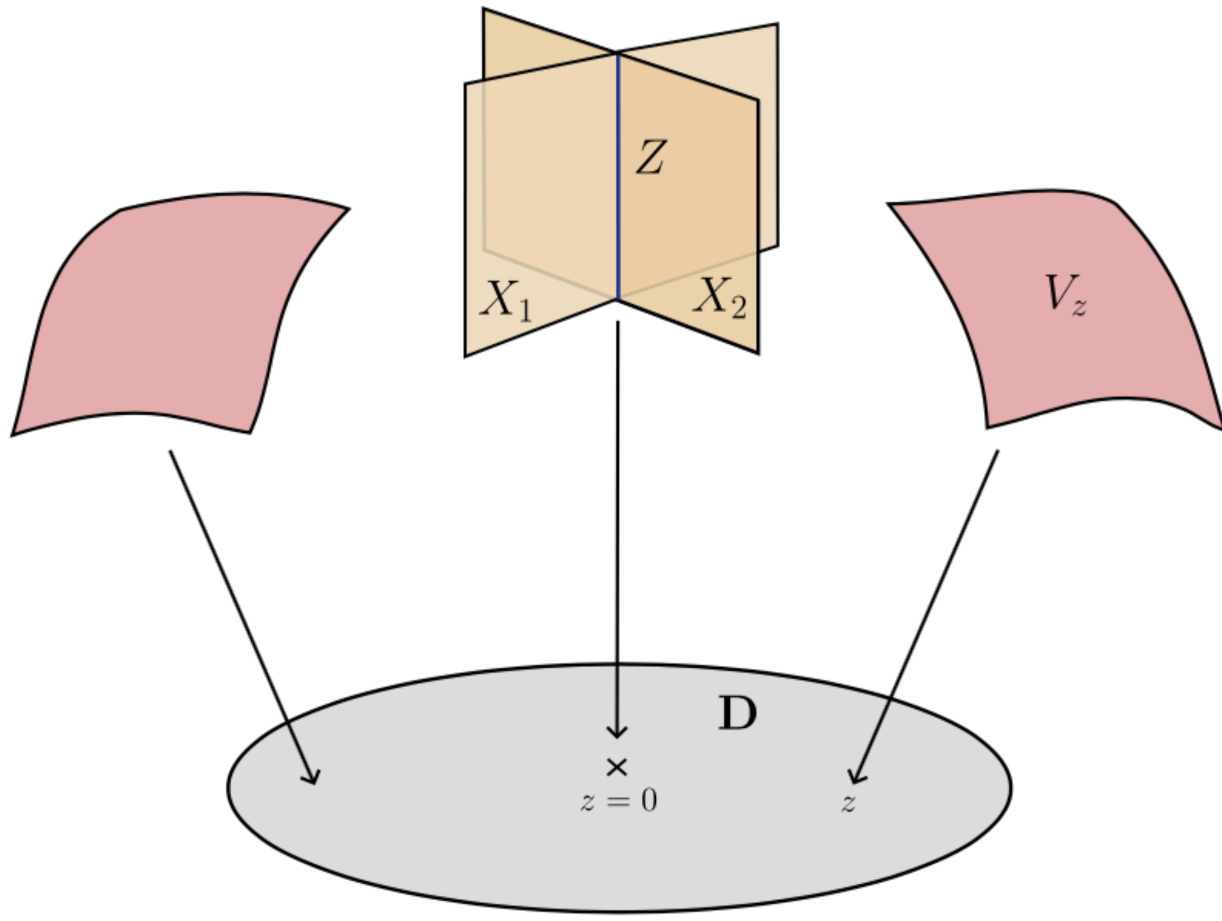
ESC in Type IIB complex structure limits

- Axionic string solution in 4d N=2 supergravity is a **critical heterotic string**:

  Reduction of IIB 4-form along localised 2-forms at intersection Z

  gives string zero modes of $(c_L, c_R) = (24,12)$

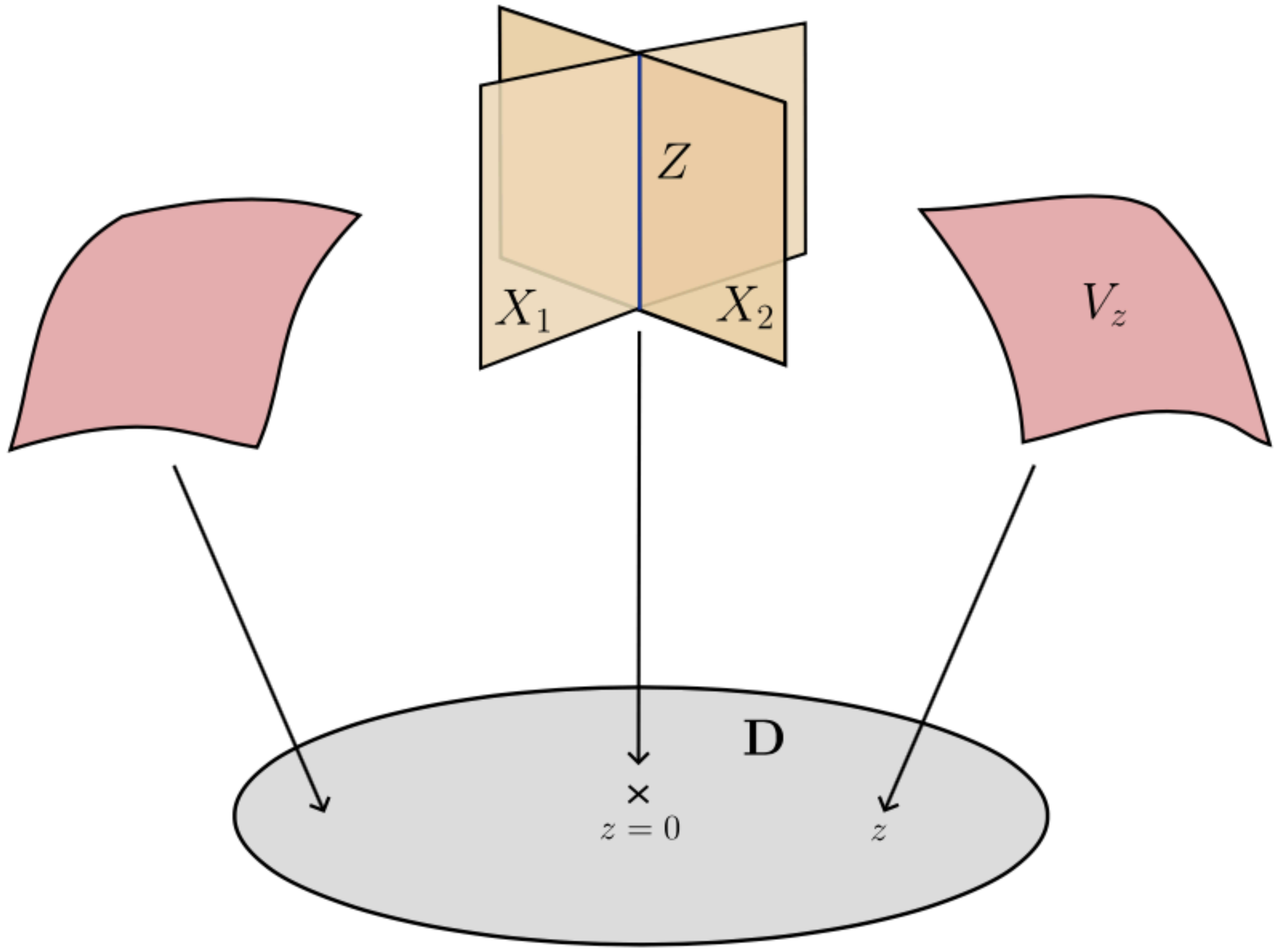
  $\Longrightarrow$ critical heterotic string!



- Interactions identify **heterotic target space** as $T^2 \times K3_{\text{het}} \times \mathbb{R}^{1,3}$

Dual heterotic gauge algebra: $\text{rank}(G_{\text{het}}) = 2 + b = \text{rk}\Lambda_{\text{trans}}(Z)$ in II_b degeneration

ESC in Type IIB complex structure limits



- Class of **special Lagrangian 3-cycles** Γ_a locally S^1 fibered

$$\Gamma_a \simeq S^1 \times C_a \qquad C_a \text{ curve on } Z$$

Tower of particles from **multi-wrapped D3-branes** exists if

$$C_a \cdot_Z C_a \geq 0 \Longleftrightarrow g(C_a) \geq 1$$

$\Longrightarrow$ Tower becomes asymptotically light at same rate as string excitations

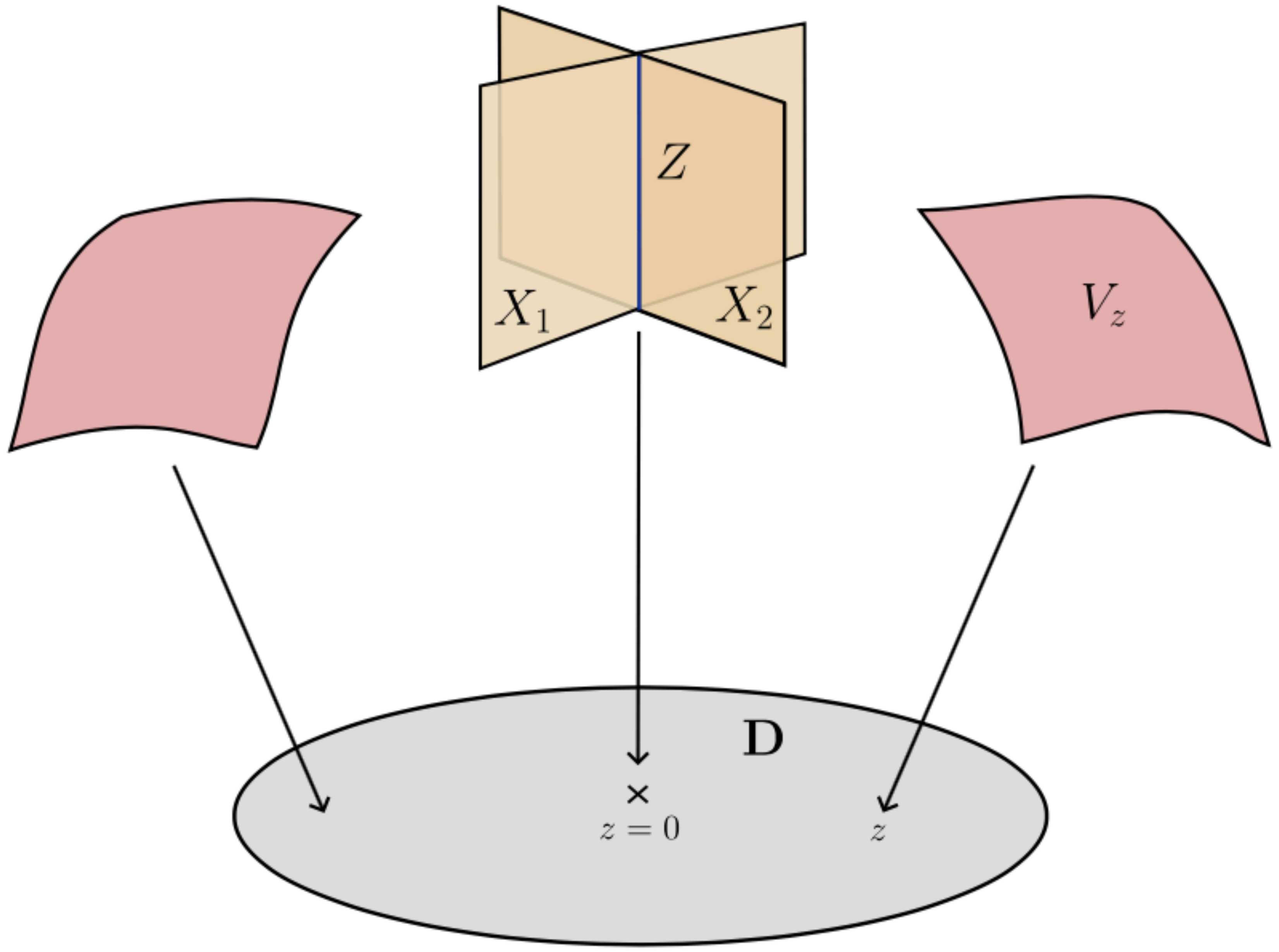
Interpreted as **KK/winding tower of dual heterotic string** at heterotic string scale

ESC in Type IIB complex structure limits

✓ ESC explicitly verified for such Tyurin limits (of type II) **[Friedrich, Monnee, TW, Wiesner'25]**

Imposing ESC further motives **new math conjectures**:

1) Every type II limit is of generalised Tyurin type (several intersections, all Abelian or all K3 of same polarisation)



θ : (at worst mock-) modular

2) New proposal for counting of sLag 3-cycles Γ_a:

$$\theta(q) = \sum_{n \in \mathcal{J}} c(n) q^n \qquad \Omega_{\text{BPS}}(\Gamma_a) = c\left(\frac{1}{2} C_a \cdot_Z C_a\right)$$

cf. [Banerjee,DiPietro,Longhi'22]

Summary

Bottom-up / IR

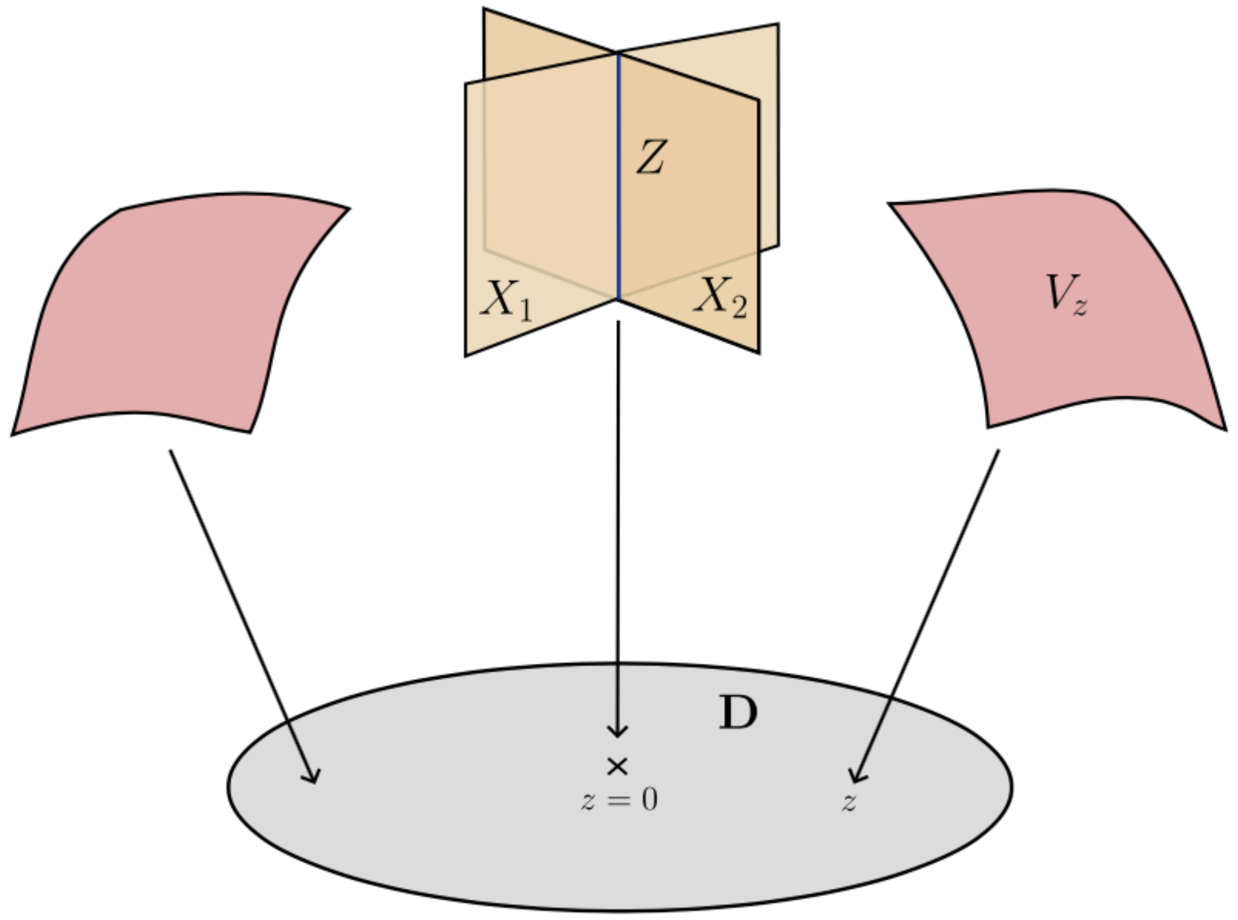
Probe branes as a promising tool for EFT constraints:

Every 5d N=1 supergravity theory **with a non-compact vector multiplet moduli space** is either a 6d theory compactified on S^1 to 5d or a Type II/heterotic string theory.

Top-down / UV

New geometric analysis of Type IIB complex structure degenerations :

Novel way to derive Type II-heterotic duality



APPENDIX

Limits: Class A

Claim: Given a Kähler (sub)cone with

1) $\mathscr{F}_{000} = 0\,,$

2) $\mathscr{F}_{00i} \neq 0$ for $i \in \mathscr{I}_1,$

3) $\mathscr{F}_{00r} = 0$ for $r \in \mathscr{I}_3\,,$

then: $\mathscr{F}_{0rs} = 0$

Proof:

SUGRA string with $p^I = \delta^I_0$ with 't Hooft anomaly matrix $k(A^I, A^J) = \mathscr{F}_{0IJ}$ of signature $(1{,}r)$

[Katz,Kim,Tarazi,Vafa'20]

Fix **diagonal basis** $\tilde{A}^0,$ $\tilde{A}^a,$ $\tilde{A}^\zeta$ where

$$k = (+1, \quad -1,\ldots,-1, \quad 0,\ldots,0)$$

Expand: $$A^0 = a_0\tilde{A}^0 + a_\alpha\tilde{A}^\alpha + a_\zeta\tilde{A}^\zeta, \quad A^r = b^r_0\tilde{A}^0 + b^r_\alpha\tilde{A}^\alpha + b^r_\zeta\tilde{A}^\zeta$$

$\Longrightarrow$ **1)** $0 = \mathscr{F}_{000} = k(A^0, A^0) = a_0^2 - a_\alpha a_\alpha\,,$ **3)** $0 = \mathscr{F}_{00r} = k(A^0, A^r) = a_0 b^r_0 - a_\alpha b^r_\alpha$

Limits: Class A

SUGRA string with $p^I = \delta^I_0$ with 't Hooft anomaly matrix $k(A^I, A^J) = \mathscr{F}_{0IJ}$ of signature $(1,r)$

[Katz,Kim,Tarazi,Vafa'20]

Expand: $A^0 = a_0\tilde{A}^0 + a_\alpha\tilde{A}^\alpha + a_\zeta\tilde{A}^\zeta, \quad A^r = b^r_0\tilde{A}^0 + b^r_\alpha\tilde{A}^\alpha + b^r_\zeta\tilde{A}^\zeta$

$\Longrightarrow$ **1)** $0 = \mathscr{F}_{000} = k(A^0, A^0) = a_0^2 - a_\alpha a_\alpha\,,$ **3)** $0 = \mathscr{F}_{00r} = k(A^0, A^r) = a_0 b^r_0 - a_\alpha b^r_\alpha$

$$0 \leq \mathscr{F}_{0rr} = k(A^r, A^r) = (b^r_0)^2 - b^r_\alpha b^r_\alpha \overset{\text{3)}}{=} \frac{(a_\alpha b^r_\alpha)^2}{a_0^2} - b^r_\alpha b^r_\alpha \overset{\substack{\text{Cauchy-Schwarz}\\ \downarrow}}{\leq} \frac{(a_\alpha a_\alpha)(b^r_\beta b^r_\beta)}{a_0^2} - b^r_\alpha b^r_\alpha \overset{\text{1)}}{=} 0$$

$\Longrightarrow \mathscr{F}_{0rr} = 0$ and **Cauchy-Schwarz is saturated**

$\Longrightarrow$ On space $\mathscr{B}$ where k has maximal rank $A^0|_{\mathscr{B}} = \beta_{(r)} A^r|_{\mathscr{B}}\,,$ $\beta_{(r)} \neq 0$ otherwise $\mathscr{F}_{00I} = 0 \quad \forall I$

Limits: Class A

SUGRA string with $p^I = \delta^I_0$ with 't Hooft anomaly matrix $k(A^I, A^J) = \mathscr{F}_{0IJ}$ of signature $(1,r)$

[Katz,Kim,Tarazi,Vafa'20]

Expand: $A^0 = a_0\tilde{A}^0 + a_\alpha \tilde{A}^\alpha + a_\zeta \tilde{A}^\zeta, \quad A^r = b^r_0 \tilde{A}^0 + b^r_\alpha \tilde{A}^\alpha + b^r_\zeta \tilde{A}^\zeta$

$\Longrightarrow \mathscr{F}_{0rr} = 0$ $\Longrightarrow A^0|_{\mathscr{B}} = \beta_{(r)} A^r|_{\mathscr{B}}\,, \quad \beta_{(r)} \neq 0$ on space $\mathscr{B}$ where k has maximal rank

Similarly: $A^0|_{\mathscr{B}} = \beta_{(s)} A^s|_{\mathscr{B}}\,, \quad \beta_{(s)} \neq 0$ for any A^s with $\mathscr{F}_{00s} = 0$

$$0 = \mathscr{F}_{00r} = k(A^0, A^r) = a_0 b^r_0 - a_\alpha b^r_\alpha = \beta_{(s)}\left(b^s_0 b^r_0 - b^s_\alpha b^r_\alpha\right) = \beta_{(s)} k(A^s, A^r) = \beta_{(s)} \mathscr{F}_{0rs}$$

$\Longrightarrow \mathscr{F}_{0rs} = 0$