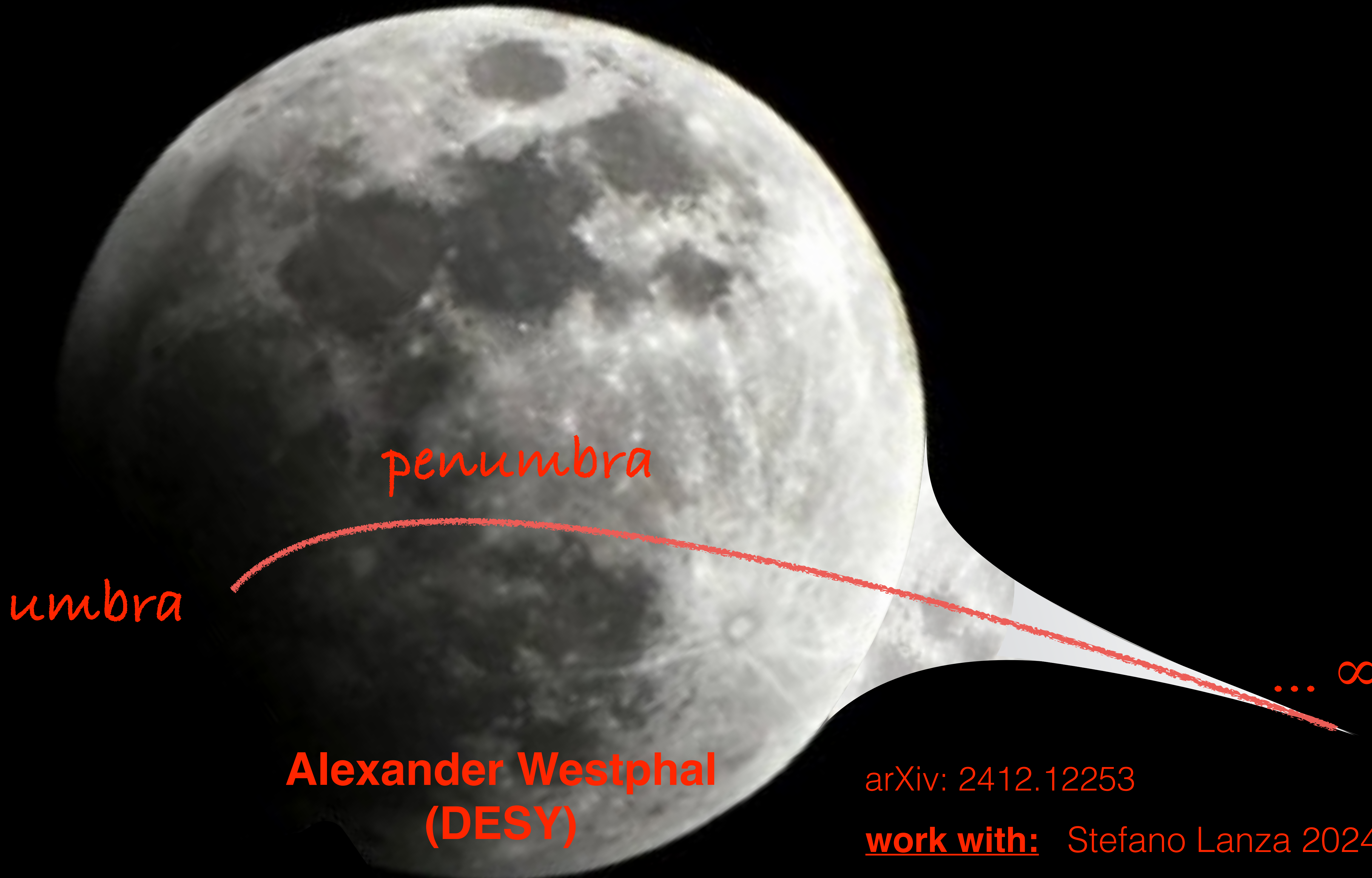
# Uplifts in the Penumbra:
# Features of the Moduli Potential just before Infinity

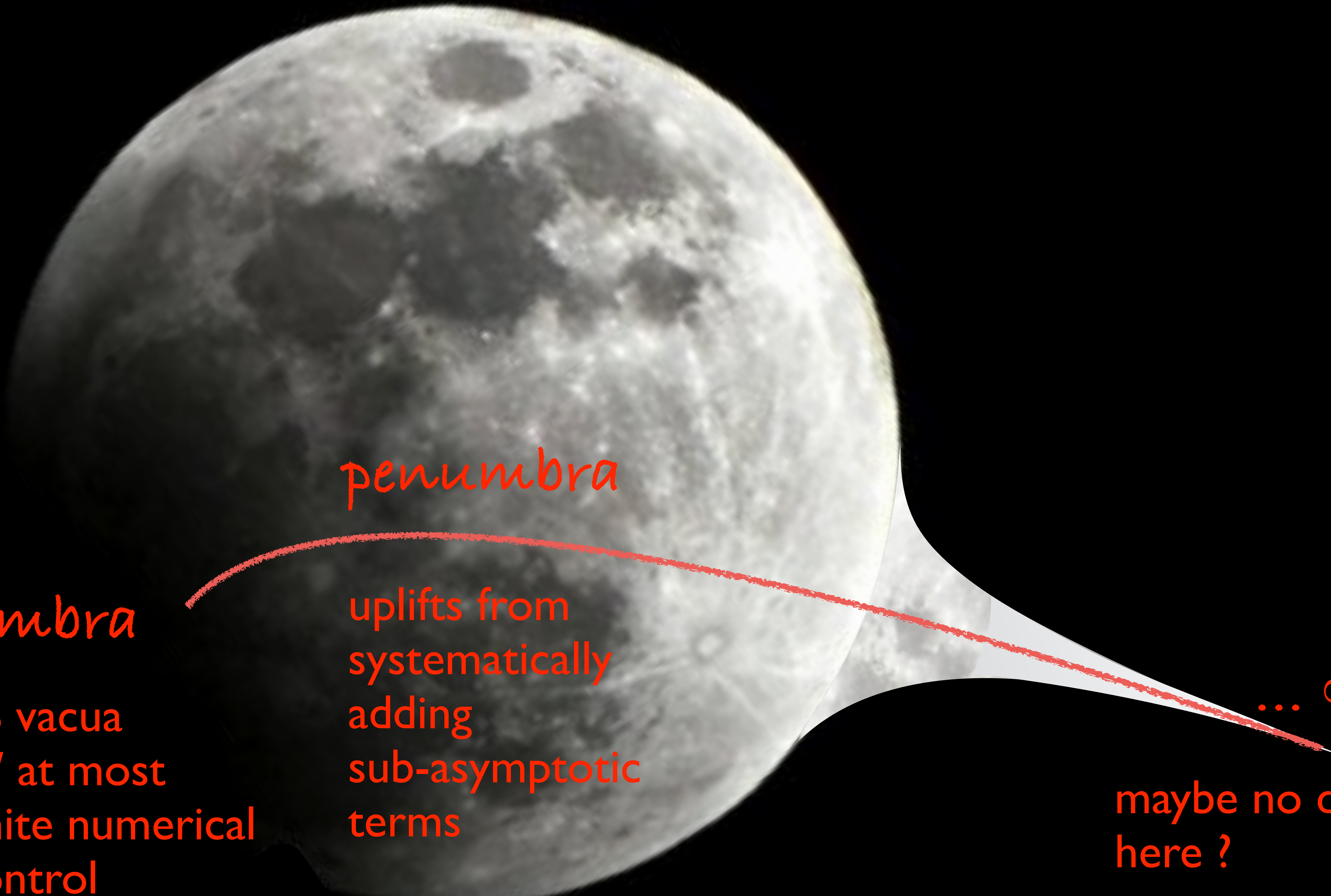



**Alexander Westphal**
**(DESY)**

arXiv: 2412.12253

**work with:** Stefano Lanza 2024

# IIB on CY — complex structure moduli space



# Moduli stabilization into meta-stable de Sitter

*see talks by Arthur, Irene, Miguel, Severin !*

- … stick with type IIB strings on Calabi-Yau orientifolds

- (i) 3-form fluxes fix the c.s. moduli & the axio-dilaton
  — no-scale in the Kähler moduli

[GKP '01]

- (ii) non-/perturbative corrections fix Kähler moduli

[KKLT, LVS, ……….]

(K) F-term breaking
Kähler stabilization

(C) F-term breaking
local c.s. minima

- (iii) uplift to dS

(A) add-on uplift
source (anti-D3 , …)

# Goal ... search for uplifts of type (C)

- only step (i):

  - fluxes freeze c.s. moduli & axio dilaton
  - volume moduli remain flat - 4D supergravity is no-scale

$$K = \hat{K} + K_{cs} = -2\ln\mathcal{V} - \ln(-i(\tau - \bar{\tau})) + K_{cs}$$

$$K^{\rm cs} = -\log \mathbf{i}(\bar{X}^I \mathcal{F}_I - X^I \bar{\mathcal{F}}_I)$$

$$W(z,\tau) = \int \Omega \wedge G_3 = \mathbf{g}^T \eta \mathbf{\Pi}(z) \ \ , \ \ G_3 = \mathbf{g}\gamma = (\mathbf{f} - \tau\mathbf{h})\gamma$$

$$\mathbf{\Pi}(z) = \begin{pmatrix} X^I(z) \\ -\mathcal{F}_I(z) \end{pmatrix} \qquad z_j = a_j + i s_j \ , \ j = 1 \ldots h^{2,1}_{-}$$

# Goal ... search for uplifts of type (C)

- only step (i):

  - fluxes freeze c.s. moduli & axio dilaton
  - volume moduli remain flat - 4D supergravity is no-scale

$$V_F = e^K (K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2)$$

$$= e^{\hat{K}} e^{K_{cs}} K_{cs}^{i\bar{\jmath}} D_i W D_{\bar{\jmath}} \bar{W}$$

only $F_3$ flux:
non-ISD = non-SUSY

$$= e^{\hat{K}} \underbrace{\mathbf{f}^T \mathcal{T}(z,\bar{z})\,\mathbf{f}}_{V^{\rm cs}}\,, \qquad \text{with} \quad \mathbf{f} = \begin{pmatrix} m^I \\ -e_I \end{pmatrix}$$

$$\mathcal{T}(a,s) \sim e^{K_{cs}} (D\Pi \bar{D}\bar{\Pi} + \Pi\bar{\Pi})$$

# Goal ... search for uplifts of type (C)

- look for F-term breaking minima in the c.s. moduli potential

$$V_F^{\rm flux} \sim \frac{1}{\mathcal{V}^2} V^{\rm cs} \quad , \quad \langle V^{\rm cs} \rangle \geq 0 \quad \Rightarrow \quad \text{Uplift}$$

originally proposed by: [Saltman & Silverstein '04]

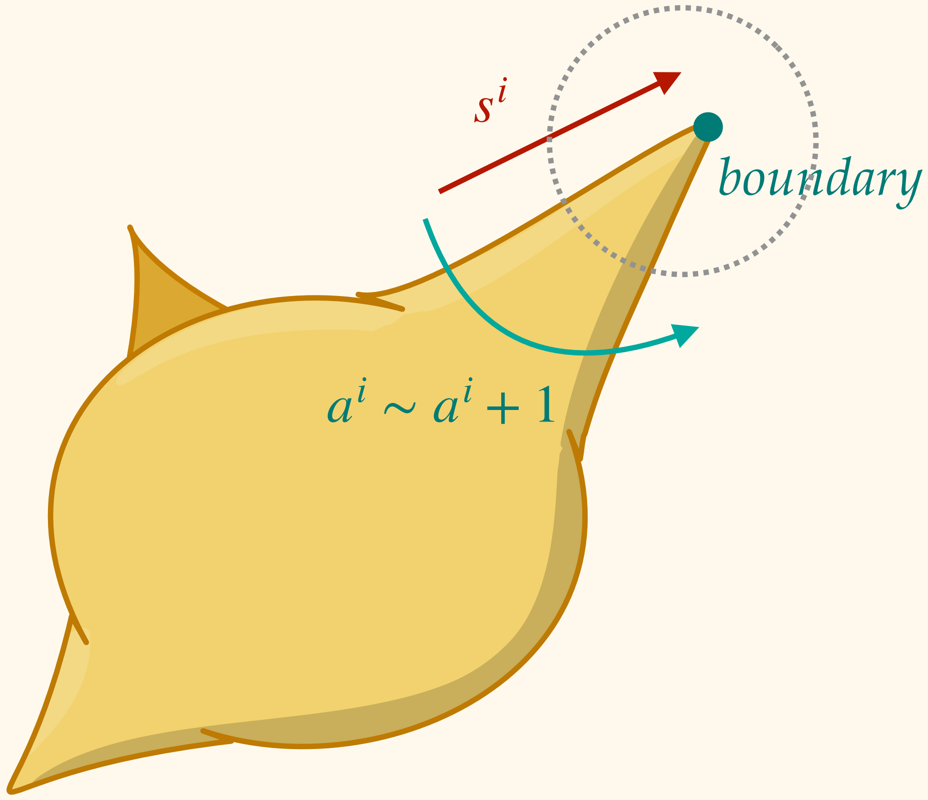
found in continuous flux approx: [Gallego, MCD Marsh, Vercnocke & Wrase '17]

in the context of 'winding' c.s. valleys from GV-controlled non-perturb. effects: [Hebecker & Leonhardt '20] [Carta, Mininno, Righi & AW '21]

and in explicit CY construction near Large Complex Structure (LCS) point of c.s. moduli space: [Krippendorf & Schachner '23]

# idea: construct c.s. potential near infinite-distance

⇒ **Use Hodge theory to find near-boundary expressions for the periods**




monodromy of period transformations at infinite distance boundary

# idea: construct c.s. potential near infinite-distance

**⇒ Use Hodge theory to find near-boundary expressions for the periods**

[Grimm, Li & Valenzuela 2020]

- convenient choice of coordinates

$z^i$ (complex) $\rightarrow$ $a^i$: axions, $s^i$: saxions (real)

- approximate $\mathbf{\Pi}(z)$ via nilpotent orbit theorem

[Schmid, 1973]
[Cattani, Kaplan, Schmid, 1986]

$$\mathbf{\Pi}(z) \simeq \mathbf{\Pi}_{\text{nil}} := e^{zN}\mathbf{a}_0$$

$N$ with $N^{n+1} = 0$

Additional geometric data

$$\mathbf{\Pi}(a,s) \xrightarrow{a \to a+1} \mathbf{\Pi}'(a,s) =: T\mathbf{\Pi}(a,s)$$

$$T := e^N$$

Corrections to the nilpotent orbit approximation are $\mathbf{a}_1 e^{-2\pi s} + \mathbf{a}_2 e^{-4\pi s} + \dots$

# idea: construct c.s. potential near infinite-distance

⇒ **Use Hodge theory to find near-boundary expressions for the periods**

**[Grimm, Li & Valenzuela 2020]**

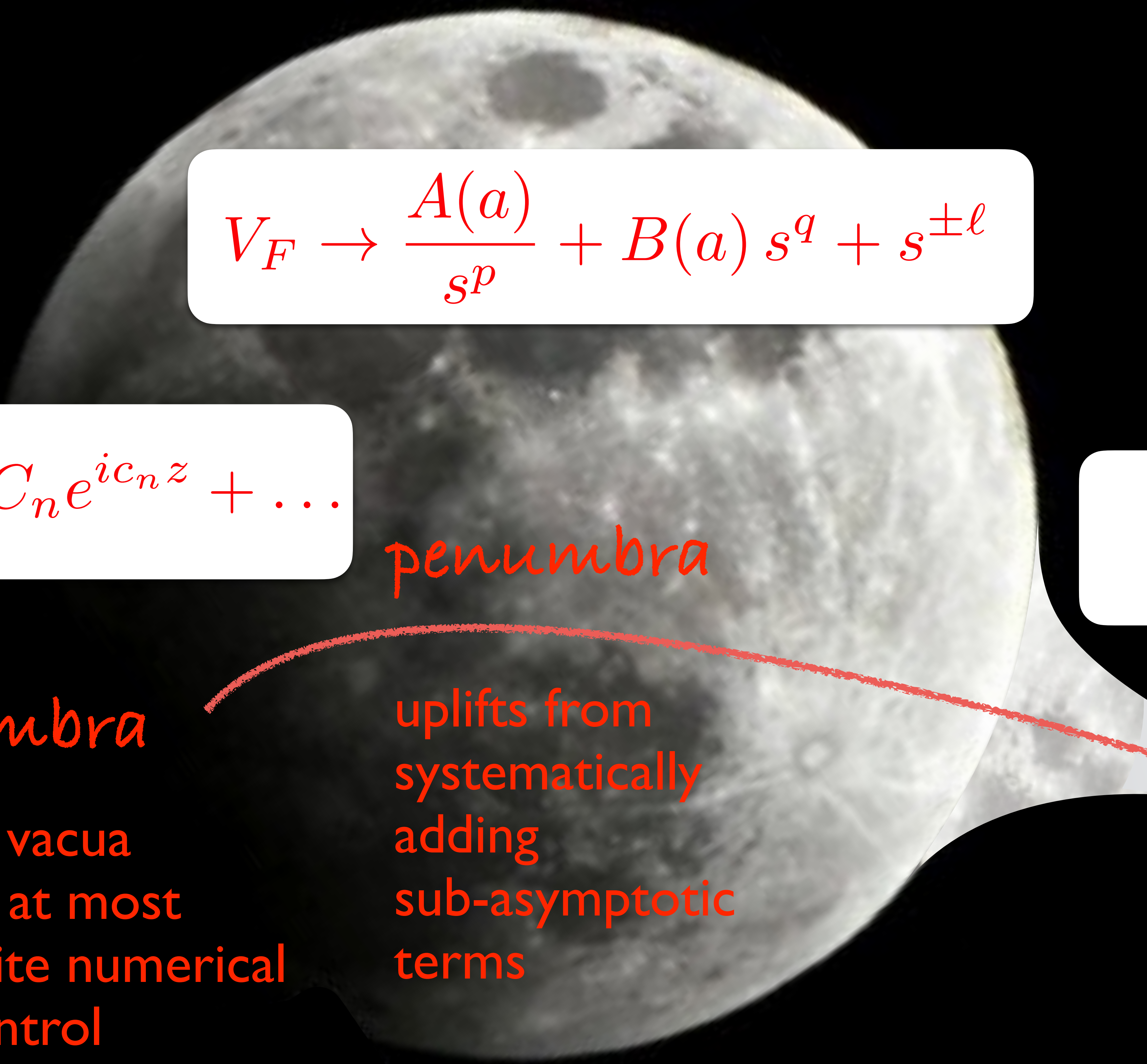
$$V_F = e^{\hat{K}} \mathbf{f}^T \mathcal{T}(z, \bar{z}) \mathbf{f}$$

↙

$$V^{\text{nil}} = e^{\hat{K}} \, \rho^T(a) \, \mathcal{Z}(s) \, \rho(a) \, , \qquad \text{with} \quad \rho(a) = e^{-aN} \mathbf{f}, \quad \mathbf{f} = \begin{pmatrix} m^I \\ -e_I \end{pmatrix}$$

- from now just one c.s. modulus $z$

# complex structure moduli space



[Grimm, Li, Valenzuela, 2020]
[Grimm 2021; Grimm, Lanza & Li, 2022]

consistent with tame geometry & Hodge theory expectations

$$V_F \to \frac{A(a)}{s^p} + B(a)\, s^q + s^{\pm\ell}$$

$$+ \sum C_n e^{i c_n z} + \ldots$$

$$V_F \to s^{\pm\ell}$$

penumbra

umbra

dS vacua
w/ at most
finite numerical
control

uplifts from
systematically
adding
sub-asymptotic
terms

... ∞

maybe no dS
here ?

# idea: construct c.s. potential near infinite-distance

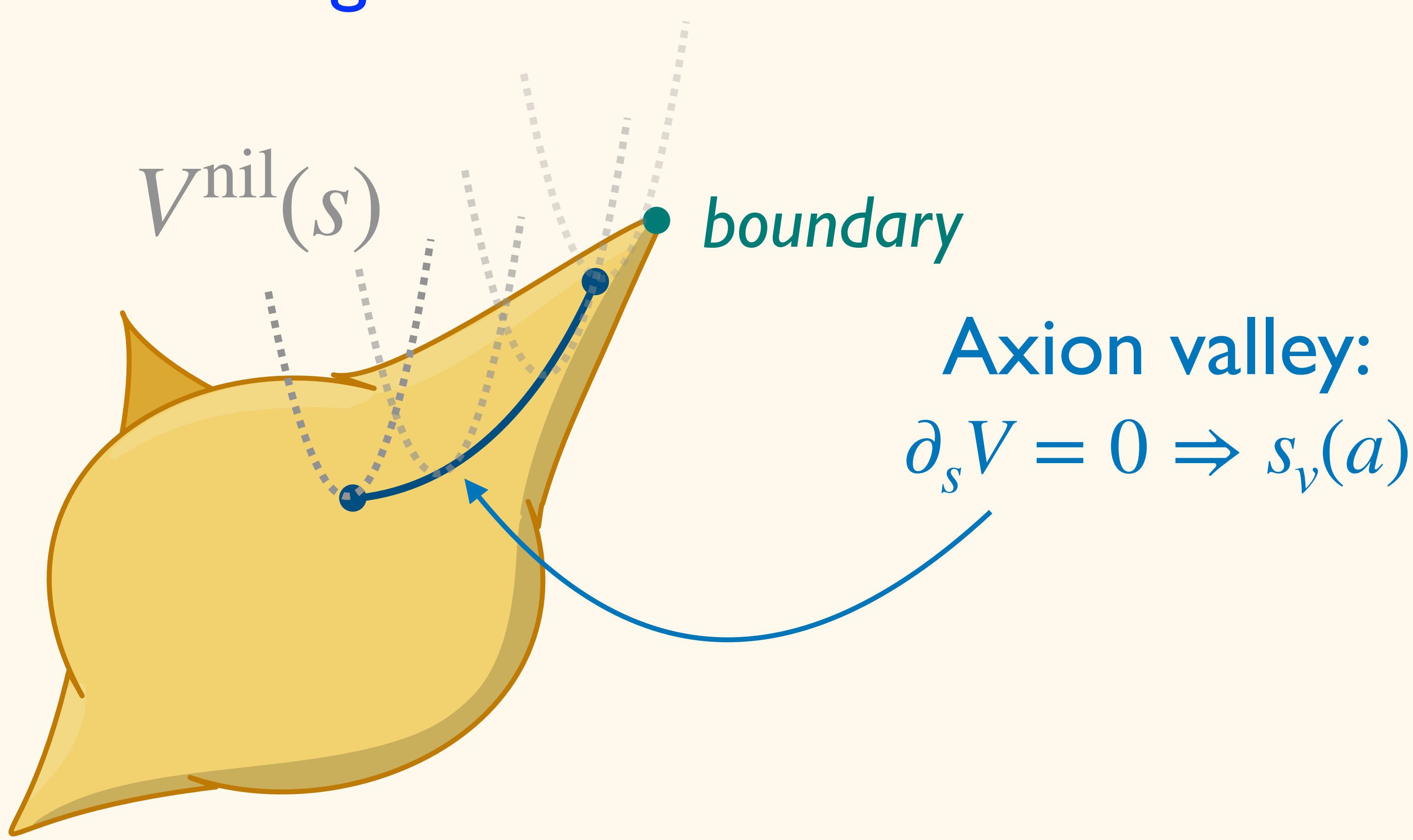
- penumbra: we can look for minima with $V_{\text{cs}} > 0$

- and for non-periodic axion valleys with moduli backreaction

  — axion monodromy —

  [McAllister, Silverstein & AW 2008]
  [Kaloper & Sorbo 2008]

  & check if they get flat enough for inflation




early works:
[Dong, Horn, Silverstein & AW 2011]
[Hebecker, Kraus & Witkowski 2014]
[Hebecker, Mangat, Rompineve & Witkowski 2014]

# asymptotic swampland constraints on dS

- **de Sitter conjecture:**

[Obied, Ooguri, Spodyneiko & Vafa, 2018]
[Hebecker & Wrase, 2018]
[Garg & Krishnan 2018]

$$\gamma = \frac{|\nabla V|}{V} \geq c_d \quad \text{with } |\nabla V|^2 := G^{AB}\partial_A V \partial_B V,\ c_d \sim \mathcal{O}(1)$$

*de Sitter coefficient*

- **Strong de Sitter conjecture:**

[Rudelius, 2018]

$$\gamma = \frac{|\nabla V|}{V} \geq c_d \quad \text{with } c_d = \frac{2}{\sqrt{d-2}}$$


- No de Sitter vacuum **asymptotically in the moduli space** — see [Grimm, Li, Valenzuela, 2020], [Calderón-Infante, Ruiz, Valenzuela, 2022];
- Slow-roll inflation **asymptotically** forbidden, as

$$\varepsilon = \frac{\gamma^2}{2\left(1 + \frac{\Omega^2}{(d-1)^2 H^2}\right)} \overset{geodesic}{\simeq} \frac{\gamma^2}{2} \gtrsim \mathcal{O}(1)$$

# testing the dS conjecture in the penumbra

- The asymptotic scalar potential is

$$V^{\rm nil} = e^{\hat{K}}\,\rho^T(a)\,\mathcal{Z}(s)\,\rho(a)\,, \qquad \text{with} \quad \rho(a) = e^{-aN}\mathbf{f}$$

*How to detect allowed inflationary regions?*

- Uplift de Sitter coefficient:

$$\gamma^{\rm cs}_{\rm uplift} = \frac{\sqrt{2K^{z\bar{z}}_{\rm cs}\partial_z V^{\rm flux}\partial_{\bar{z}}V^{\rm flux}}}{V^{\rm flux}}$$

- Late de Sitter coefficient:

$$\gamma^{\rm cs}_{\rm late} = \frac{\sqrt{2K^{z\bar{z}}_{\rm cs}\partial_z V^{\rm flux}\partial_{\bar{z}}V^{\rm flux}}}{V^{\rm flux} - V^{\rm flux}_{\rm min}}$$

*to fake full mod. stab. at* $V_{\rm min} \simeq 0$

⇒ Possible inflationary paths:

Axion valleys $s_v(a)$

… with enough flattening & $\varepsilon \simeq \frac{(\gamma^{\rm cs}_{\rm late})^2}{2} \lesssim 1$

# Large Complex Structure (LCS) penumbra

[Green, Griffiths, Kerr, 2008], [Grimm, Lanza, van Vuren, 2022]

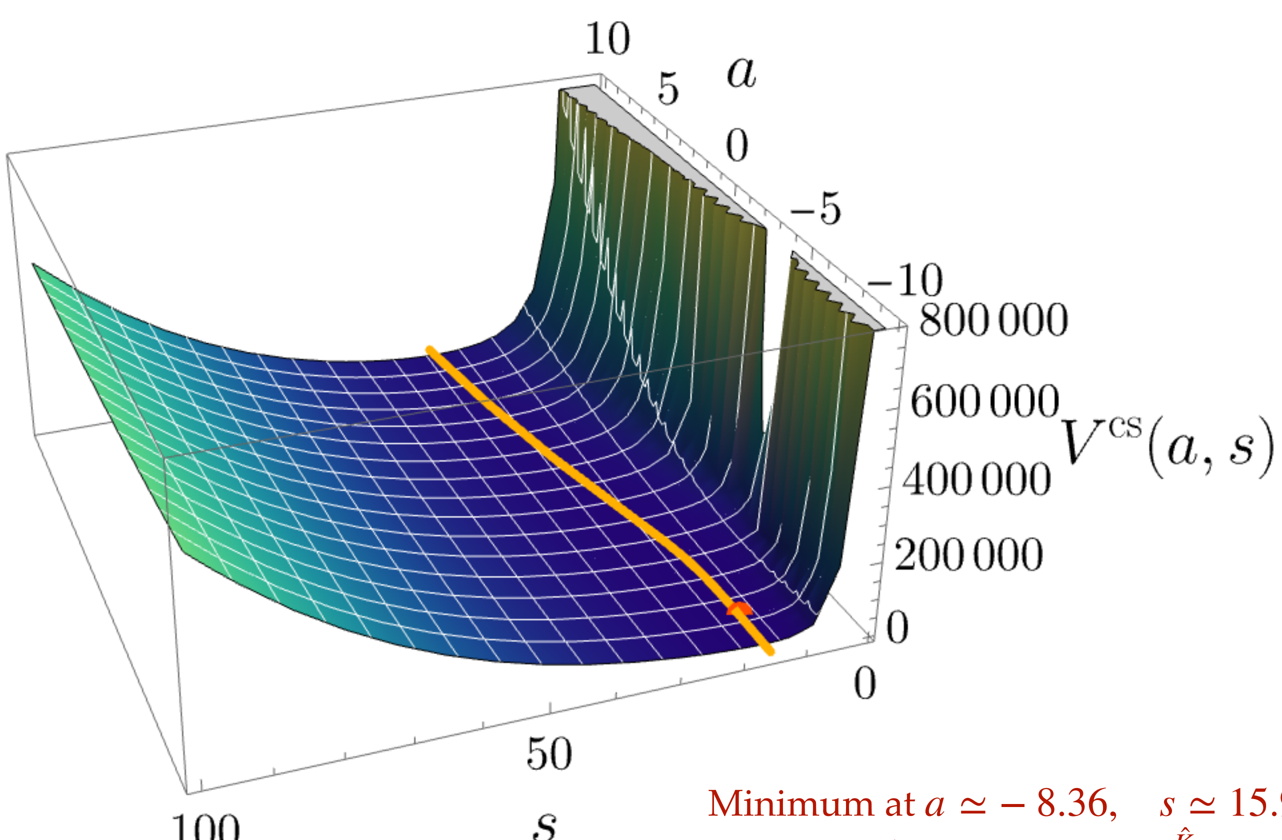
*Periods:* $\mathbf{\Pi}^{\text{nil}}_{LCS} = \begin{pmatrix} 1 \\ mz \\ -\frac{1}{6}m^2nz^3 + \frac{1}{2}cz + \xi \\ \frac{1}{2}mnz^2 + bz + \frac{c}{2m} \end{pmatrix}$

$$K^{\text{cs}}_{LCS} \simeq -\log\left(\frac{4}{3}m^2ns^3 + 2\,\text{Im}\xi\right)$$

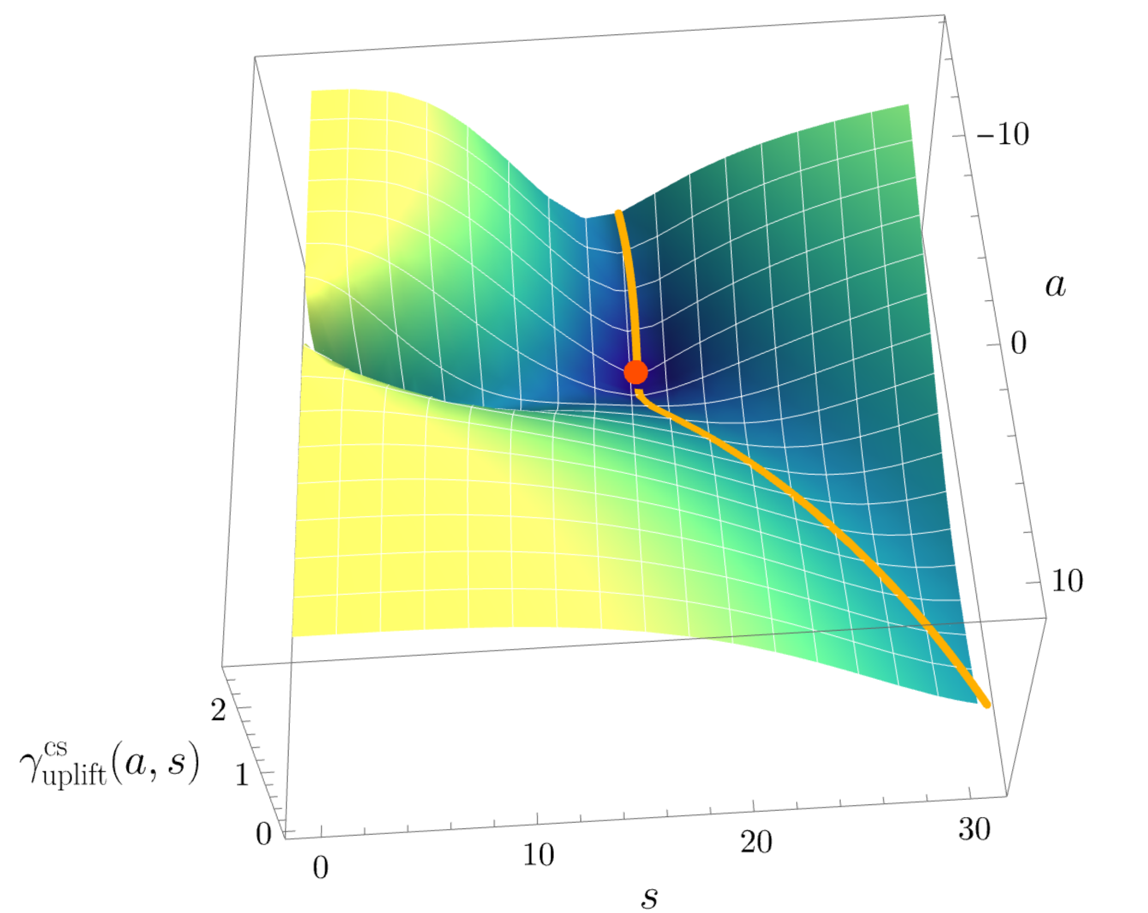
*with* $m, n \in \mathbb{Z}\,,\ m \neq 0\,,\ n > 0\,,\ b + \frac{mn}{2} \in \mathbb{Z}\,,\ c - \frac{m^2n}{6} \in \mathbb{Z}$

$V(a,s) \sim \frac{1}{s^3}(A_0a^6 + A_1a^5s + \ldots + A_6s^6)$ , $A_a = A_a\,(\,\textit{fluxes:}\ e_0, e_1, m_0, m_1\,)$

[Lanza & AW 2024]




Minimum at $a \simeq -8.36$, $s \simeq 15.9$,
with $V_{\text{min}} \simeq 18{,}756e^{\hat{K}}$



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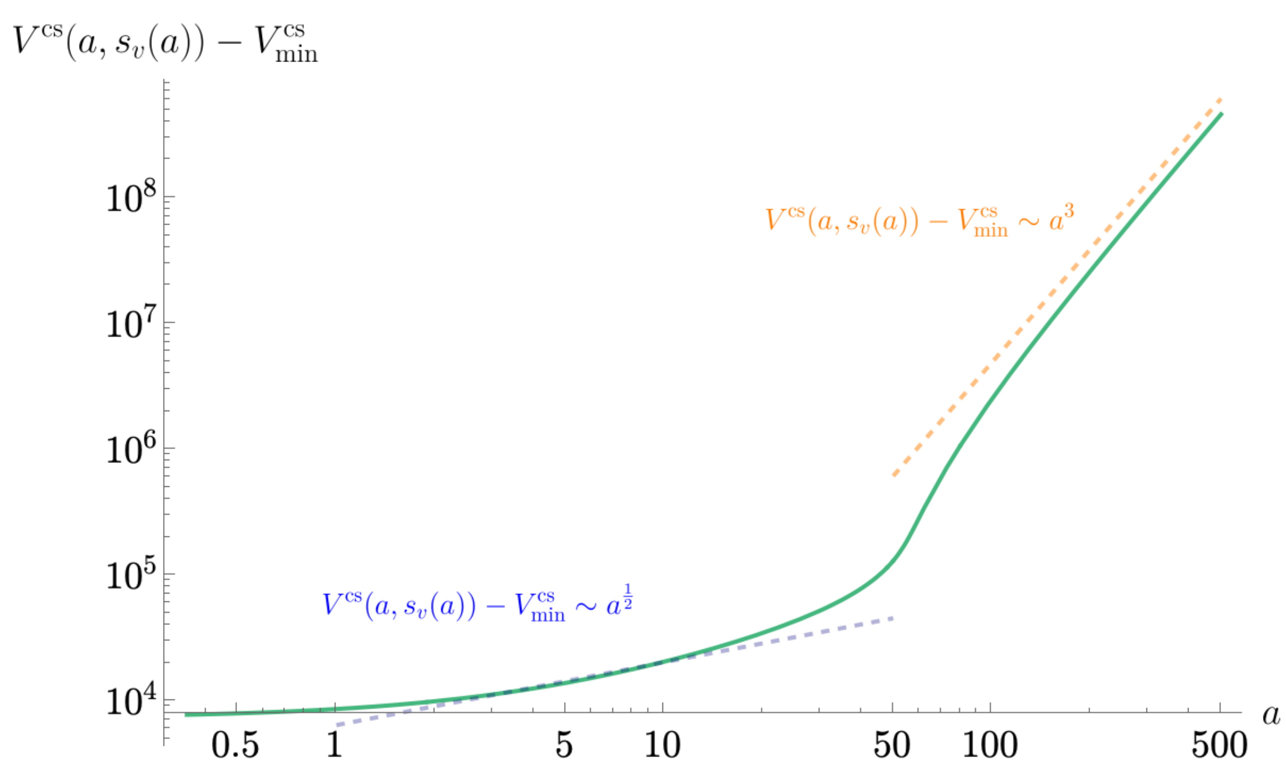
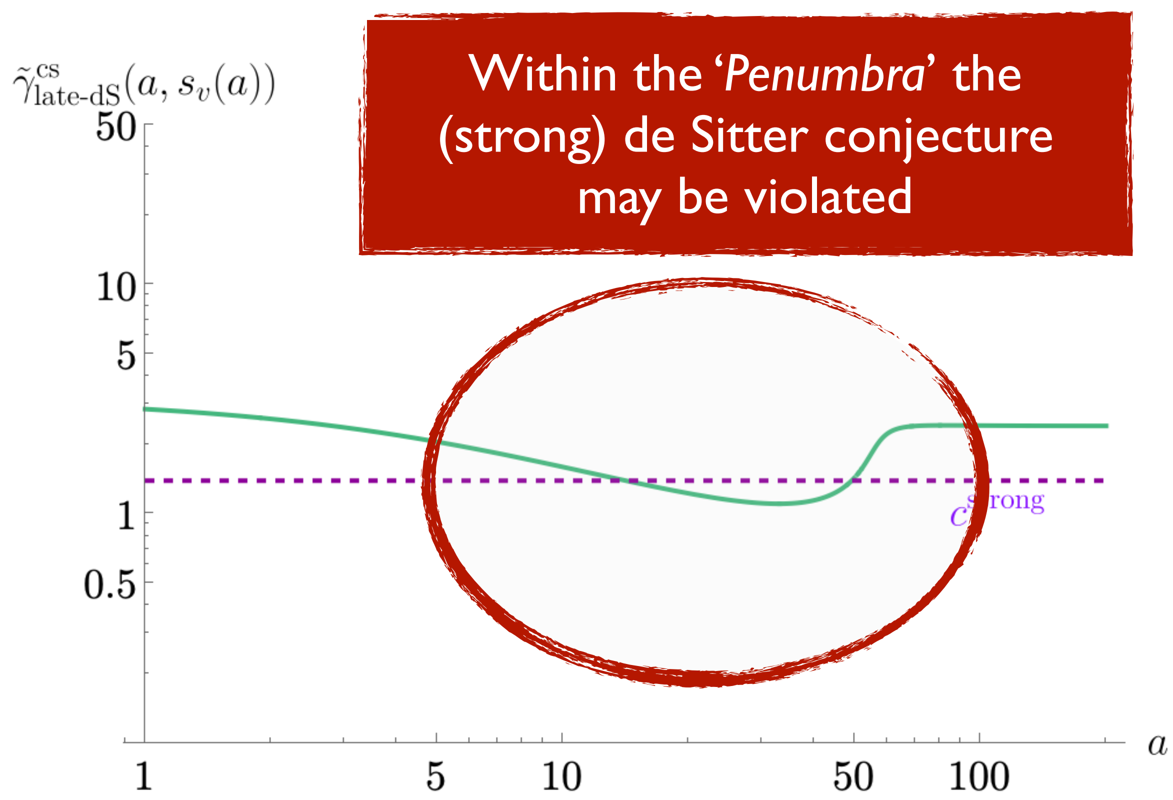


# Tyurin boundary penumbra

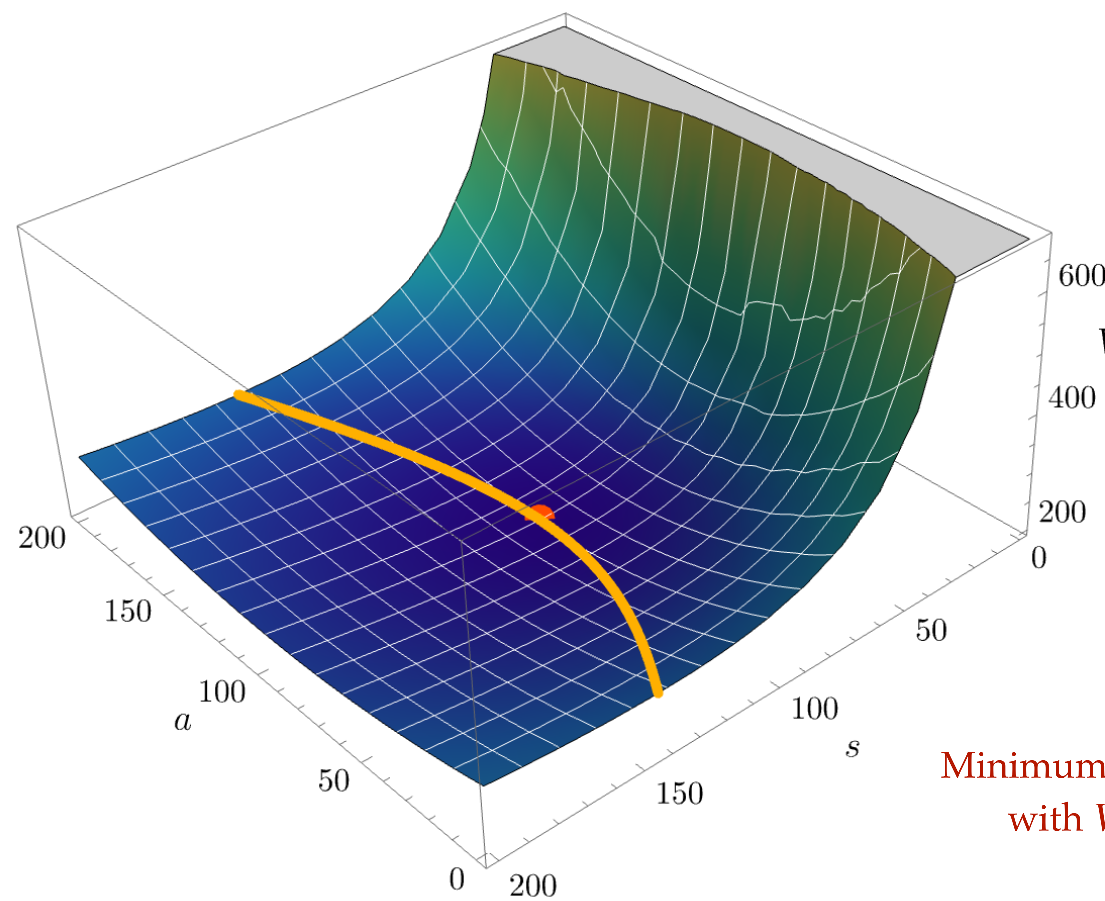
[Green, Griffiths, Kerr, 2008], [Grimm, Lanza, van Vuren, 2022]

*Periods:* $\boldsymbol{\Pi}^{\text{nil}}_{\text{Tyurin}} = \begin{pmatrix} 1 \\ \mathbf{i}\,\alpha \\ mz \\ d + \mathbf{i}\,c\alpha + \mathbf{i}\,n\alpha z \end{pmatrix}$ $\qquad K^{\text{cs}}_{\text{Tyurin}} \simeq -\log\left(4ms - 2\sqrt{\frac{m}{n}}d\right)$

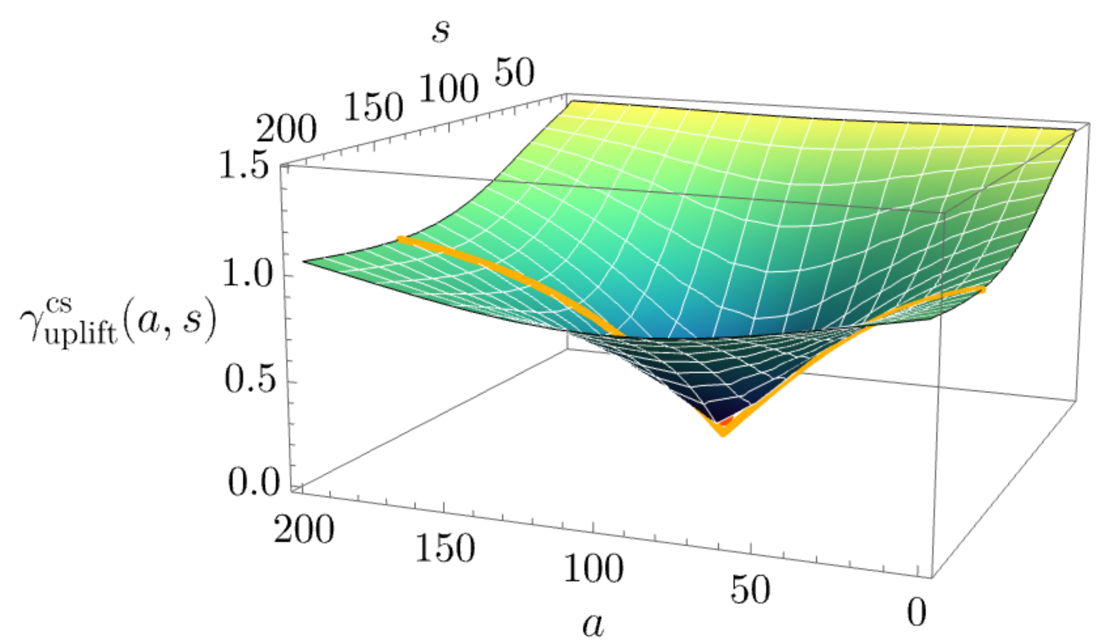
*with* $m, n \in \mathbb{N}, \quad \alpha = \sqrt{\frac{m}{n}}, \quad c, d \in \mathbb{R}$

$V(a,s) \sim \frac{1}{s}(A_0 a^2 + A_1 as + A_2 s^2) \quad , \quad A_a = A_a\,(\textit{fluxes: } e_0, e_1, m_0, m_1\,)$

[Lanza & AW 2024]




Minimum at $a \simeq s \simeq 100$,
with $V_{\text{min}} \simeq 200e^{\hat{K}}$



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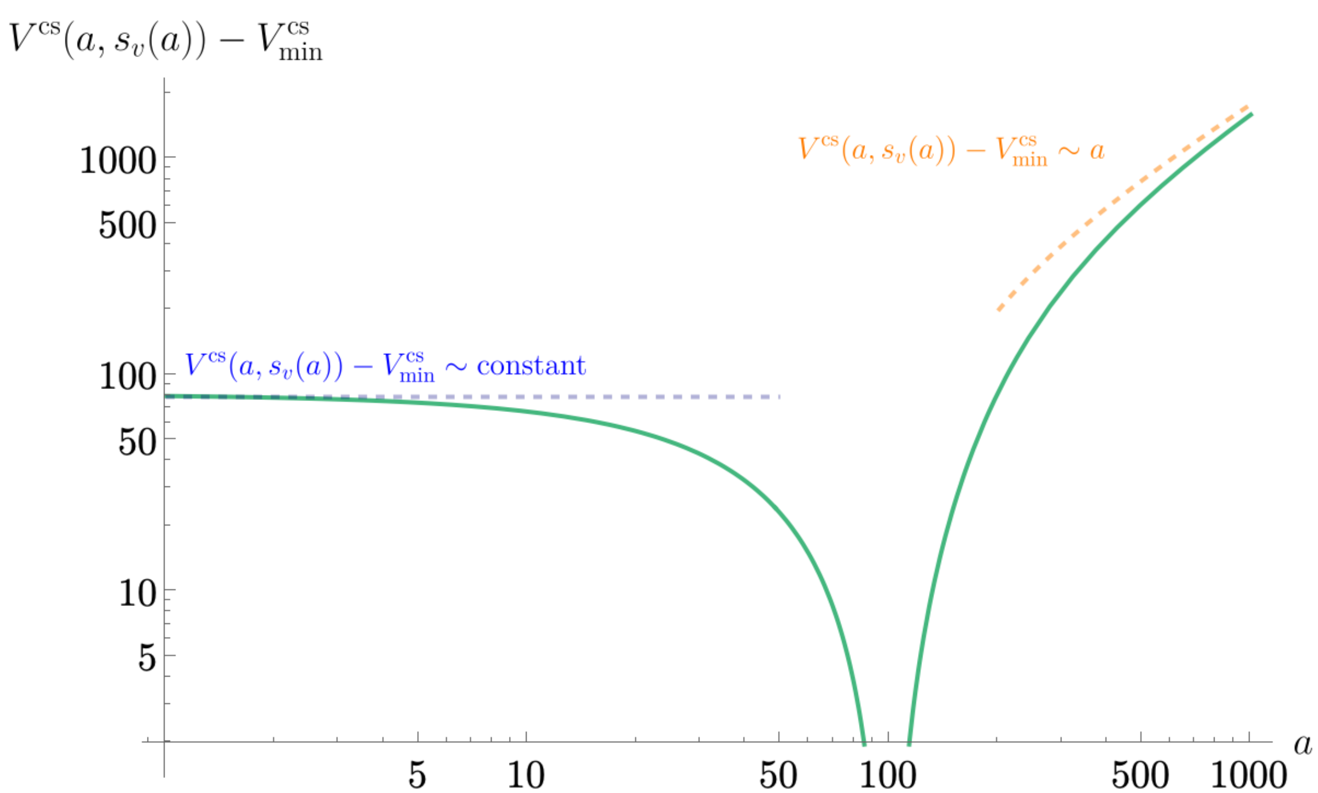


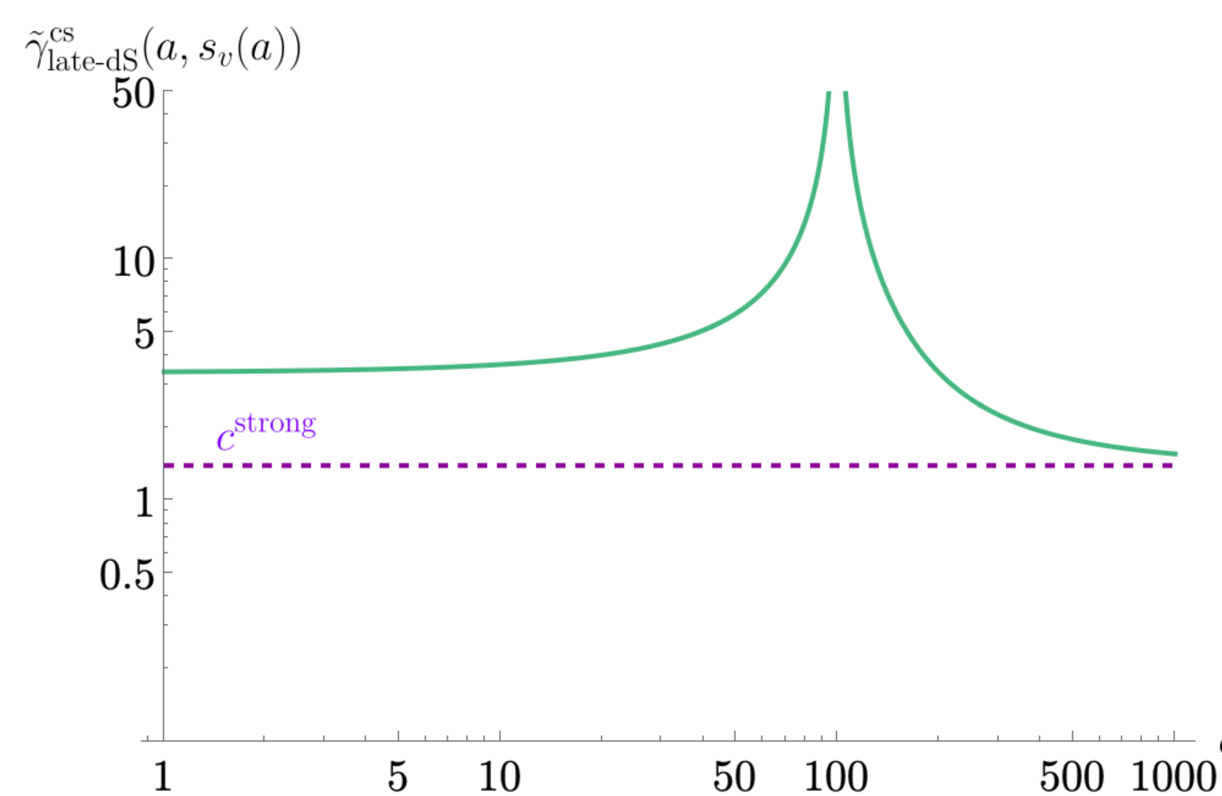

# Machine Learning good axion valleys ...

[Lanza & AW 2024]

- Models are specified by ***several*** geometric parameters and fluxes
- Different parameters can lead to substantially different physics

↓

Algorithmic searches and machine learning
to find ***parameter/field space regions*** hosting viable valleys

# summary

- systematic construction of flux potential for c.s. moduli inwards from ∞ distance boundaries — evidence for F-term SUSY breaking uplift vacua in the penumbra of c.s. moduli space

- possibly slow-roll flat curved axion-modulus valleys in the scalar potential showing axion monodromy - inflation candidates

- so far only 1 axion + 1 modulus — need to check general case

- need to combine with full moduli stabilization — are viable dS vacua possible ?

- comparison with known explicit CY examples near LCS point ?

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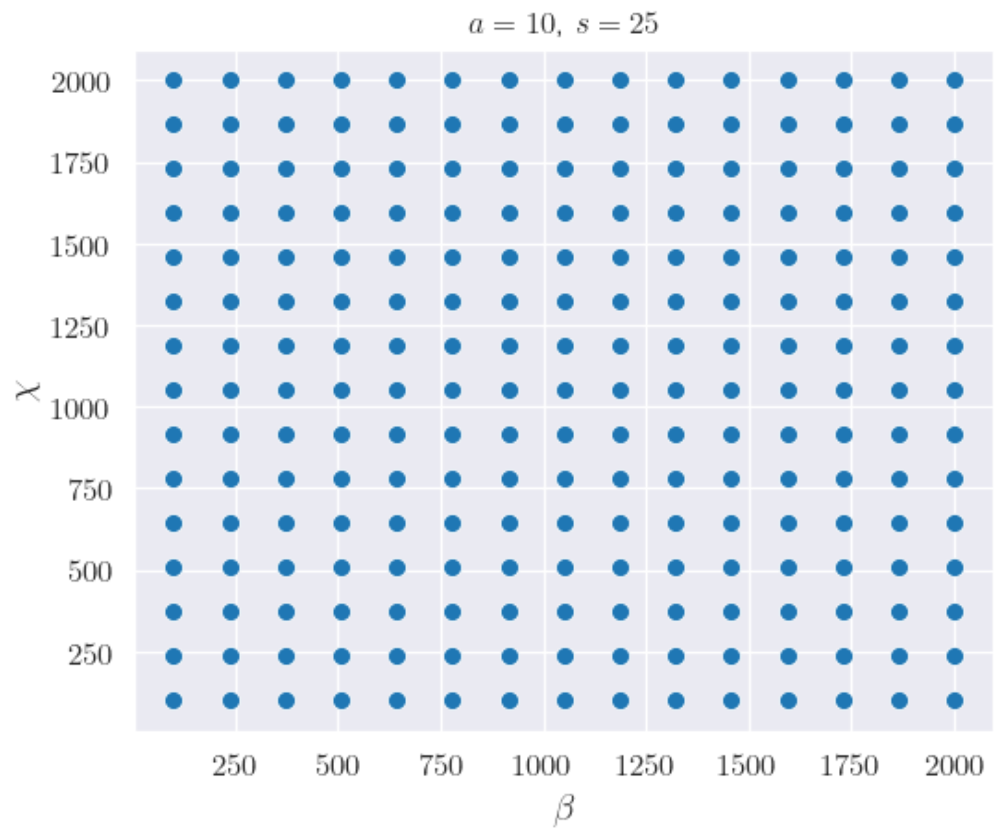
# Machine Learning good axion valleys ...

<u>An example of algorithm</u>

*Input: scalar potential* $V^{cs}(a, s; \lambda, f)$

*Goal: find parameter space regions for which slow-roll inflation is not forbidden*

1. Choose a set of field space points $(a_{(i)}, s_{(i)})$;
2. Choose ranges for parameters $\lambda$ and fluxes $\mathbf{f}$;



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<u>An example of algorithm</u>

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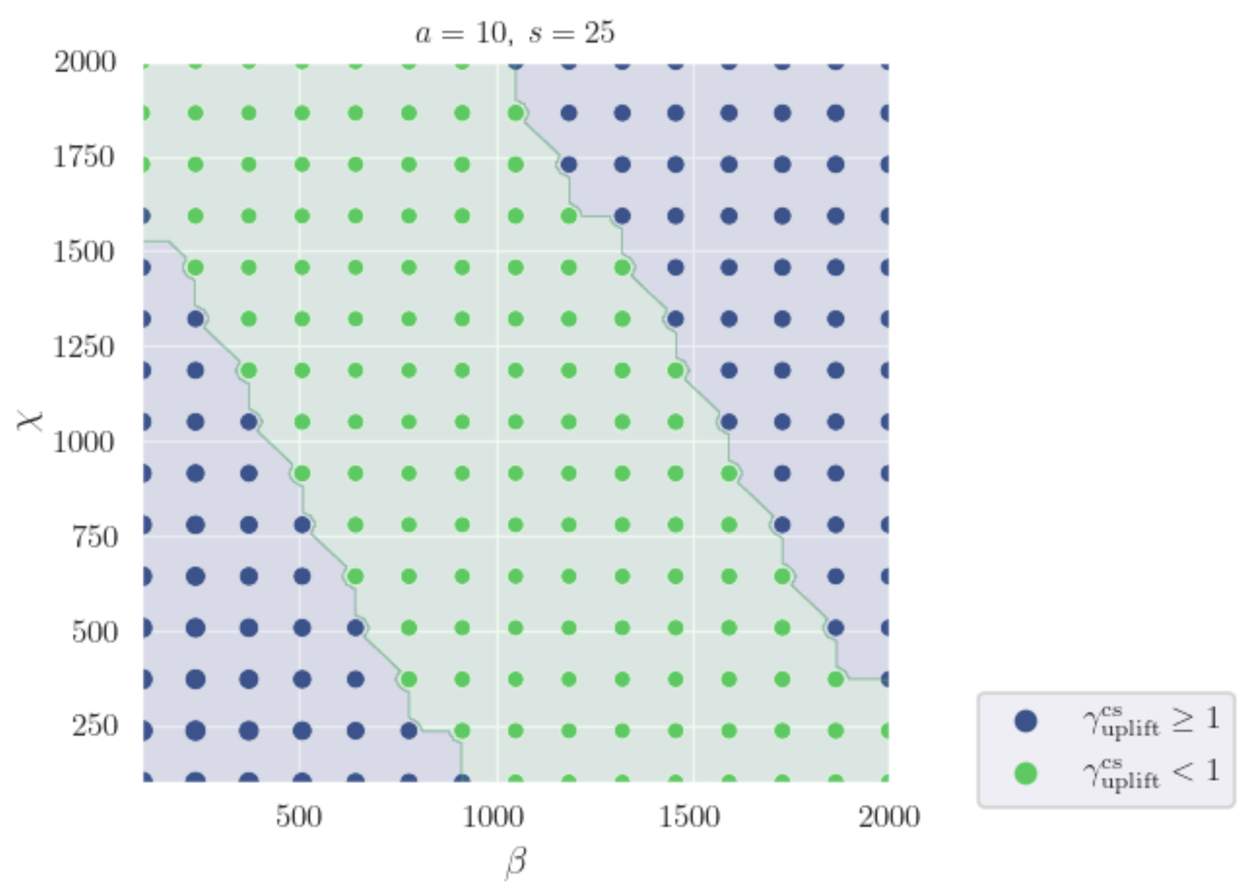
*Goal: find parameter space regions for which slow-roll inflation is not forbidden*

1. Choose a set of field space points $(a_{(i)}, s_{(i)})$
2. Choose ranges for parameters $\lambda$ and fluxes $\mathbf{f}$
3. For each field space point $(a_{(i)}, s_{(i)})$

   3a. Compute $\gamma^{cs}_{uplift}(a_{(i)}, s_{(i)}; \lambda_{(a)}, \mathbf{f}_{(a)})$ for all the parameters in the range

   3b. If, for a given $(\lambda_{(a)}, \mathbf{f}_{(a)})$, $\gamma^{cs}_{uplift}(a_{(i)}, s_{(i)}; \lambda_{(a)}, \mathbf{f}_{(a)}) < 1$ assign the label '$\mathbf{1}$', otherwise assign the label '$\mathbf{0}$'
4. Machine-learn the regions for which $\gamma^{cs}_{uplift}(a_{(i)}, s_{(i)}; \lambda_{(a)}, \mathbf{f}_{(a)}) < 1$ (e.g. *via k-nearest neighbour algorithm*)



# Machine Learning good axion valleys ...

More in general, we can scan over the parameter space, compute the valleys for each choice of the parameters, and find the minimum of $\gamma_{\rm late}^{\rm cs}$ along these valleys: