Topics in Geometric Analysis



Contribution ID: 17

Type: not specified

Non-persistence of strongly isolated singularities, and geometric applications

Tuesday, 24 June 2025 12:00 (1 hour)

In this lecture, based on recent joint work with Yangyang Li (University of Chicago) and Zhihan Wang (Cornell University), I will present a generic regularity result for stationary integral *n*-varifolds with only strongly isolated singularities inside *N*-dimensional Riemannian manifolds, in absence of any restriction on the dimension $(n \ge 2)$ and codimension. As a special case, we prove that for any $n \ge 2$ and any compact (n + 1)-dimensional manifold *M* the following holds: for a generic choice of the background metric *g* all stationary integral *n*-varifolds in (M, g) will either be entirely smooth or have at least one singular point that is not strongly isolated. In other words, for a generic metric only "more complicated" singularities may possibly persist. This implies, for instance, a generic finiteness result for the class of all closed minimal hypersurfaces of area at most $4\pi^2 - \varepsilon$ (for any $\varepsilon > 0$) in nearly round four-spheres: we can thus give precise answers, in the negative, to the well-known questions of persistence of the Clifford football and of Hsiang's hyperspheres in nearly round metrics. The aforementioned main regularity result is achieved as a consequence of the fine analysis of the Fredholm index of the Jacobi operator for such varifolds: we prove on the one hand an exact formula relating that number to the Morse indices of the conical links at the singular points, while on the other hand we show that the same number is non-negative for all such varifolds if the ambient metric is generic.

Presenter: Prof. CARLOTTO, Alessandro (Università di Trento)