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Multi-bubble isoperimetric problems

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The classical isoperimetric inequality in Euclidean space $\mathbb{R}^n$ states that among all sets of prescribed volume, the Euclidean ball minimizes surface area. One may similarly consider isoperimetric problems on more general spaces, such as on the n-sphere $\mathbb{S}^n$ and on n-dimensional Gaussian space $\mathbb{G}^n$ ($\mathbb{R}^n$ endowed with the standard Gaussian measure). Furthermore, one may consider the "multi-bubble" isoperimetric problem, in which one prescribes the volume of $k \geq 2$ bubbles (possibly disconnected) and minimizes their total surface area – as any mutual interface will only be counted once, the bubbles are now incentivized to clump together. The classical case, referred to as the single-bubble isoperimetric problem, corresponds to $k = 1$; the case $k = 2$ is called the double-bubble problem, and so on.

In 2000, Hutchings, Morgan, Ritor\'e and Ros resolved the double-bubble conjecture in Euclidean space $\mathbb{R}^3$ (and this was subsequently extended to $\mathbb{R}^n$) – the standard double-bubble, whose boundary consists of three spherical caps meeting at 120°-degree angles, uniquely minimizes perimeter given prescribed volumes.
A more general conjecture of J.~Sullivan from the 1990's asserts that when $k \leq n+1$, a standard k-bubble uniquely minimizes perimeter in $\mathbb{R}^n$.

Over the last years, in collaboration with Joe Neeman, we have resolved various cases of this conjecture and its analogues in more general spaces: \\
- the multi-bubble conjecture for $k \leq n$ bubbles in Gaussian space $\mathbb{G}^n$. \\
- the multi-bubble conjecture for $k \leq \min(5, n)$ bubbles in $\mathbb{R}^n$ and $\mathbb{S}^n$, e.g. the triple-bubble conjectures when $n \geq 3$, the quadruple-bubble conjectures when $n \geq 4$, and the quintuple-bubble conjectures when $n \geq 5$ (without uniqueness on $\mathbb{R}^n$ in the latter case).

In this mini course, I will present the various tools and ideas which were used to derive these results, ranging from diverse areas such as Geometric Measure Theory, Calculus of Variations, Elliptic PDE, Spectral Theory, and even a hint of Probability and Topology. I will emphasize the remaining open questions and promising directions for tackling them, which serve as fertile ground for further investigation.

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