Workshop on Radiative Corrections and Monte Carlo Simulations for e^+e^- collisions,
Scuola Normale Superiore Pisa,
May 7-9, 2025

Luminosity Measurement at modern e^+e^- Colliders

Cluster of Excellence PRISMA+
Precision Physics, Fundamental Interactions and Structure of Matter

FOR 5327
Project TFF
Project XYZ
Project JRP
Project ALP
Project LBL

Hadrons CRC 1660 Nuclei

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Normalization Techniques for absolute Cross Section Measurements

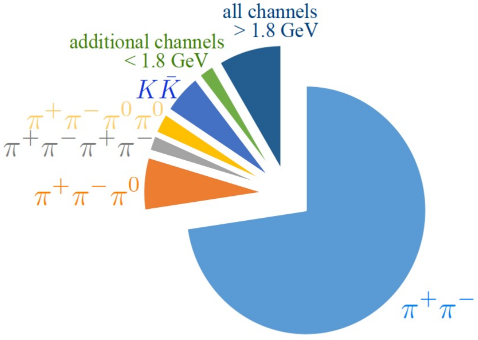
Hadronic Vacuum Polarization Contribution to $(g\text{-}2)_\mu$

JG|U

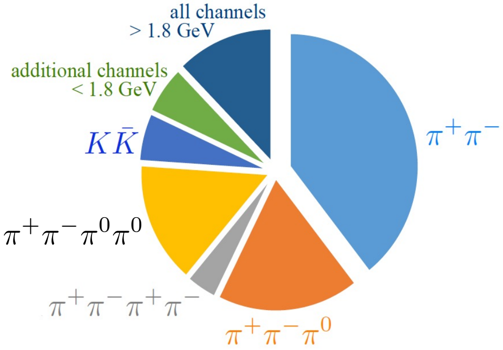
Anomalous magnetic moment of the muon $(g\text{-}2)_\mu$

$$a_\mu^{HVP} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\, \boldsymbol{\sigma_{\mathrm{had}}(s)}$$

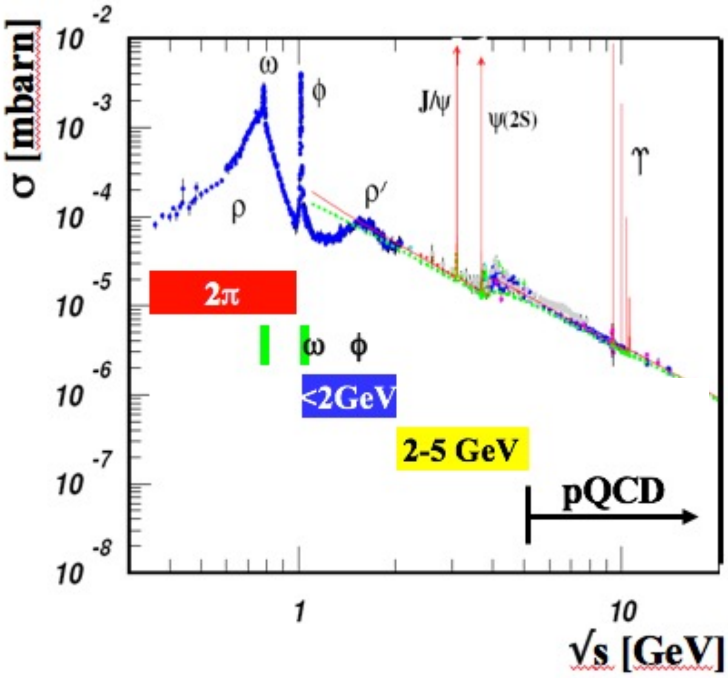
Need for high-quality cross section measurements



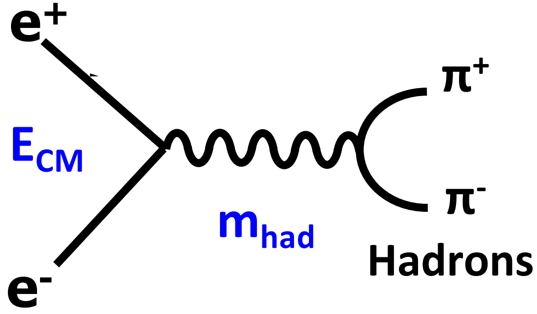
Contributions to HVP integral

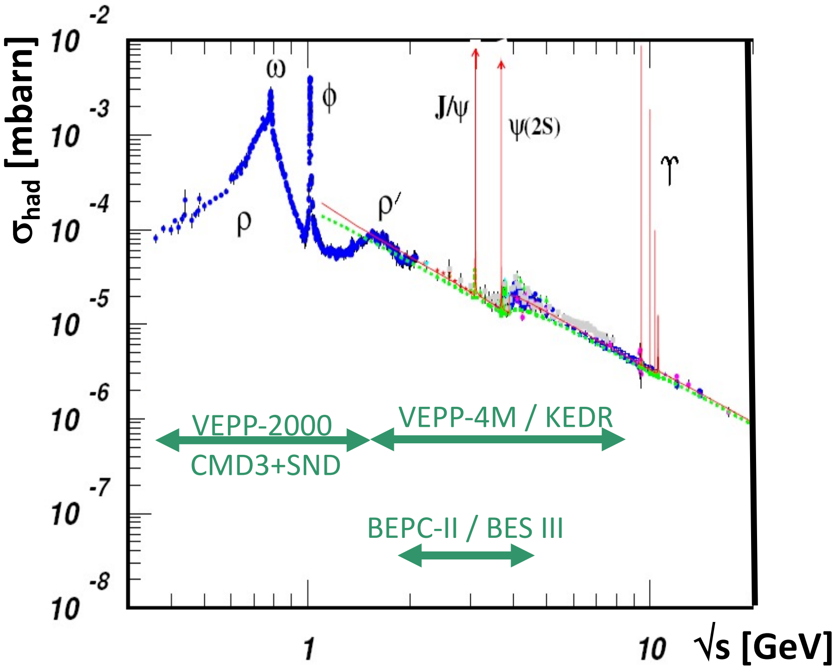


Contributions to HVP error



Measurements of σ_{had} – Energy Scan vs. Initial State Radiation





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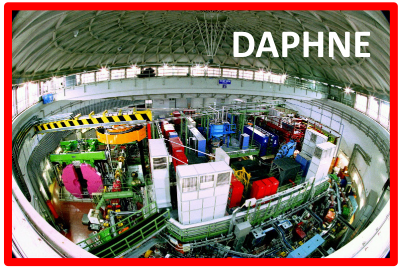
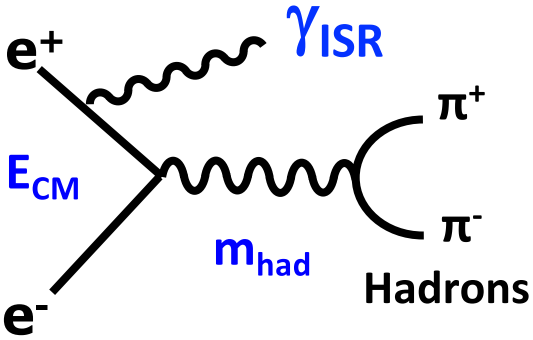
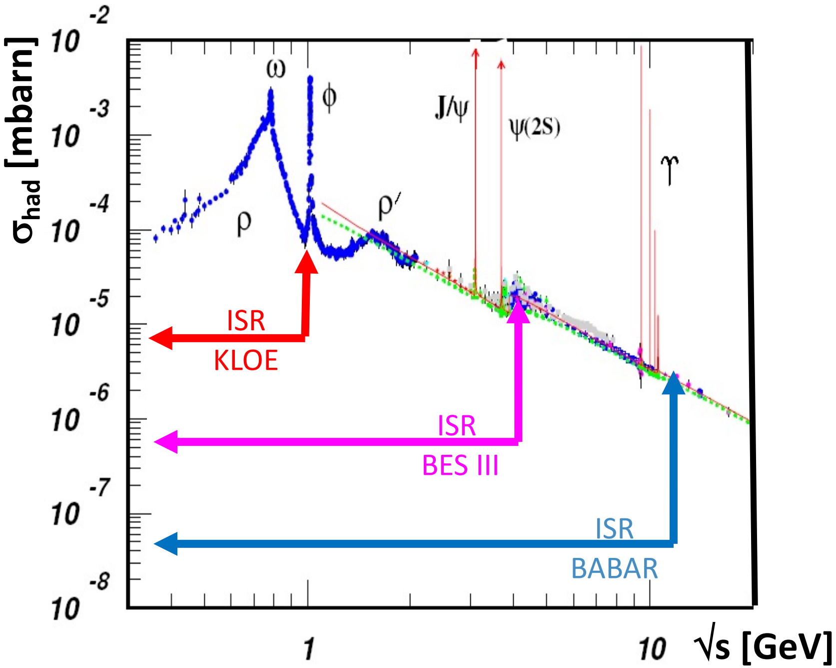






- No systematic variation of E_{beam}
- High statistics thanks to high luminosity
- Radiative corrections (H_{rad})

PHOKHARA event generator

Overview Experiments

Experiment	Published Method	Normalization
KLOE $\sqrt{s}\sim 1$ GeV	ISR untagged ISR tagged ISR untagged	Luminosity + H_{rad} Luminosity + H_{rad} μ+μ-γ
BABAR $\sqrt{s}\sim 10$ GeV	ISR tagged	μ+μ-γ
BESIII $\sqrt{s}\sim 4$ GeV	ISR tagged	Luminosity + H_{rad}
BELLE-II $\sqrt{s}\sim 10$ GeV		
CMD-2/CMD-3	Scan < ~1 GeV	e+e-
SND	Scan < ~1 GeV	e+e-

Luminosity measurement:

Normalization to QED reference channel with well-known cross section

→ Count number of signal events

$$\int \mathcal{L}\,\mathrm{dt} = \frac{N_{\mathrm{obs}} - N_{\mathrm{bkg}}}{\sigma_{\mathrm{eff}}}$$

Reference channels:

- Bhabha $e^+e^- \rightarrow e^+e^-$
- $e^+e^- \rightarrow \mu^+\mu^-$
- $e^+e^- \rightarrow \gamma\gamma$

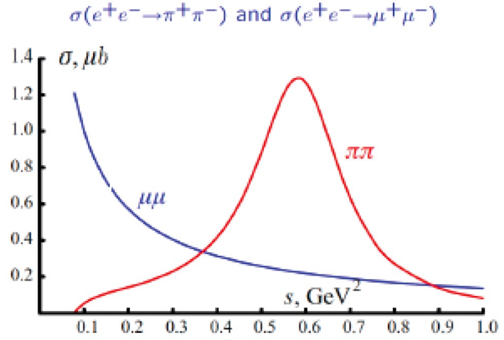
Initial State Radiation - Normalization

-

Two independent normalization methods:

1) normalization to L_{int} (obtained from Bhabha events) and H_{rad}; subtraction of background ($\mu^+\mu^-\gamma$, …)

$$\sigma_{bare}(e^+e^- \to \pi^+\pi^-) = \frac{N_{\pi\pi\gamma}/\epsilon_{exp}}{L_{int} \cdot H_{rad} \cdot \delta_{vac} \cdot (1+\delta_{FSR})}$$



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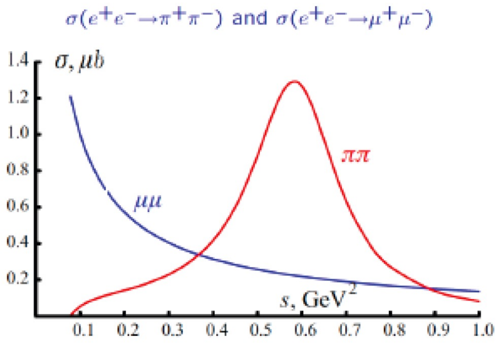
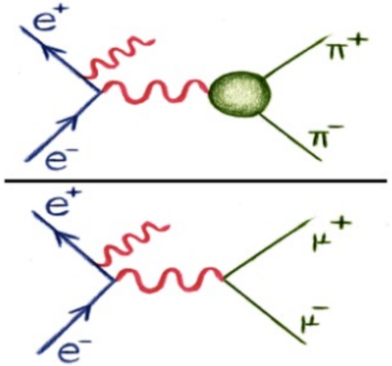


2) normalization to $\mu^+\mu^-\gamma$ events, i.e. R ratio ($\pi\pi\gamma/\mu\mu\gamma$)

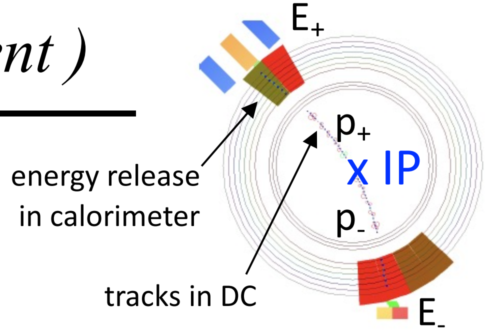
→ L_{int}, H_{rad}, δ_{vac} cancel in ratio!

→ requires high statistics of $\mu^+\mu^-\gamma$

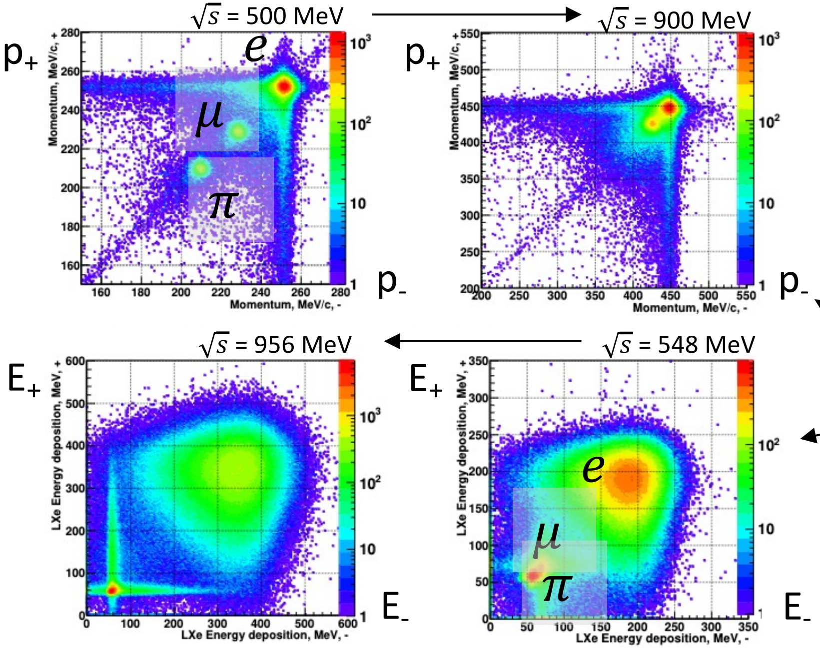
$$R = \frac{N_{\pi^+\pi^-}}{N_{\mu^+\mu^-}} \cdot \frac{\varepsilon_{\mu^+\mu^-} \cdot \left(1+\delta^{\mathrm{FSR}}_{\mu^+\mu^-}\right)}{\varepsilon_{\pi^+\pi^-} \cdot \left(1+\delta^{\mathrm{FSR}}_{\pi^+\pi^-}\right)}$$



Normalization at CMD-3 (Energy-Scan Experiment)



Normalization directly to e^+e^- via two methods



Most impressive feature: ππ/ee ratio determined independently by two complementary methods

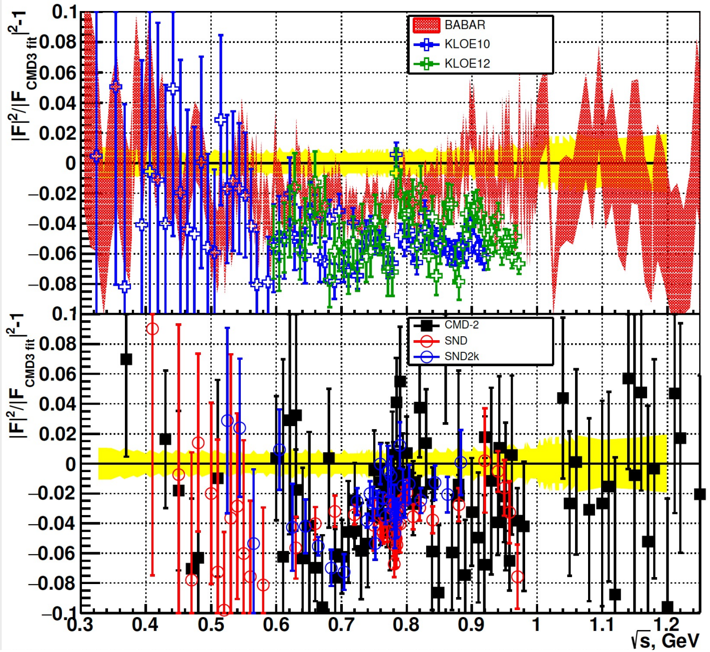
- Momentum based
- Calorimetric

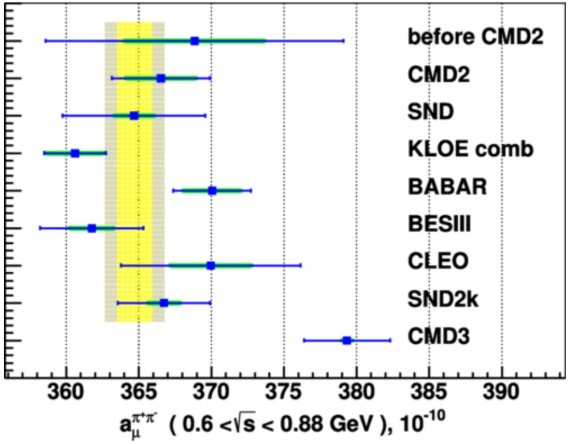
agreement ~ 0.2% around rho peak

Big Deviations seen in Channel $e^+e^- \to \pi^+\pi^-$

Unfortunately, big deviations seen between e^+e^- experiments for two-pion channel

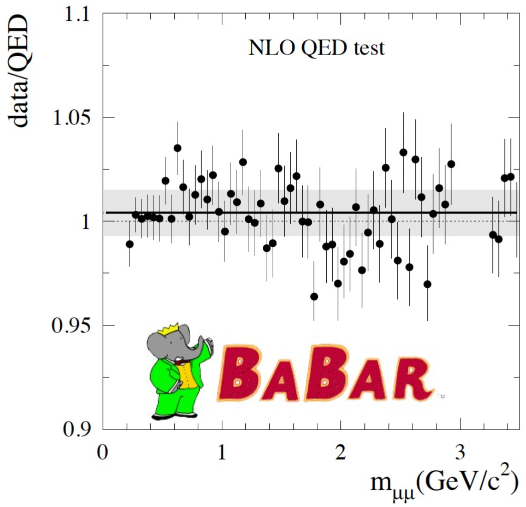
→ mistakes in luminosity measurements reason for these deviations?

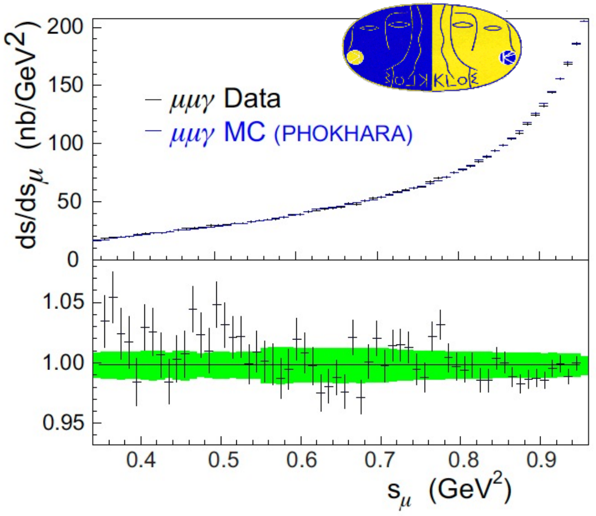


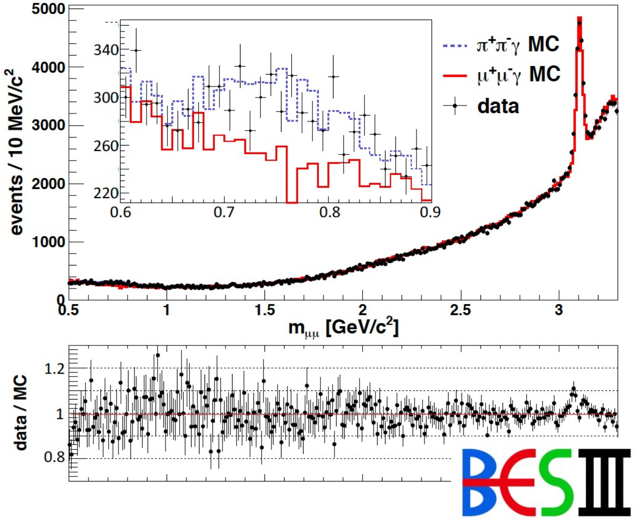


Cross Check: QED Test via $e^+e^- \to \mu^+\mu^-$

Independent cross check given by QED test
→ $e^+e^- \to \mu^+\mu^-$, however also dependence on radiator function (precision of QED generator)







Agreement <1.1%

Agreement <1.1%

Agreement <1.0%

Need for High-Quality QED Generators

For pure luminosity measurement, high-quality Bhabha generator as well as generator for di-muon and $\gamma\gamma$ - channel needed

Radiative corrections needed, however mainly Born-level selection (see below)

→ BABAYAGA@NLO serves as a high-precision tool for all three QED reference channels

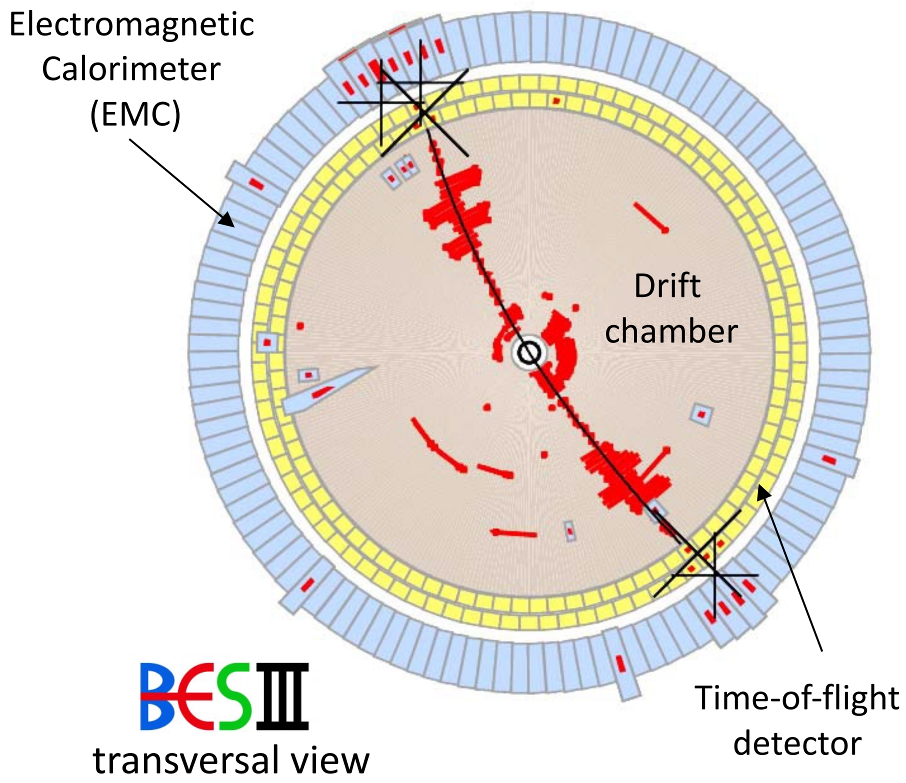
	$e^+e^- \to e^+e^-$	$e^+e^- \to \mu^+\mu^-$	$e^+e^- \to \gamma\gamma$
Order	NLOPS	NLOPS	NLOPS
Accuracy	$\mathcal{O}(0.1\%)$	$\mathcal{O}(0.1\%)$	$\mathcal{O}(0.1\%)$

Experimental Details
Luminosity Measurement

Overview Publications (likely not complete)

Experiment	Year	$\sqrt{s}$ / GeV	Control Sample(s)	Accuracy	Generator	Reference
KLOE	2006	~ 1.0	e^+e^-	0.6 %	BABAYAGA 3.5	EPJ C47 (2006) 589
BABAR	2013	~ 10.4	e^+e^-, $\mu^+\mu^-$	0.43 … 0.88 %	BABAYGA@NLO, KKMC	NIM A726 (2013) 2013
BESIII	2013	3.65, 3.773	e^+e^-	1.0 %	BABAYAGA 3.5	Chin Phys C37 (2013) 123
BESIII	2015	3.81 – 4.6	e^+e^-	1.0 %	BABAYAGA 3.5	Chin Phys C39 (2015) 093
BESIII	2017	2.23 – 4.59	e^+e^-, $\gamma\gamma$	0.7 %, 1.1 %	BABAYAGA 3.5	Chin Phys C41 (2017) 063
BESIII	2017	2.125	e^+e^-	0.8 %	BABAYAGA 3.5	Chin Phys C41 (2017) 113
BESIII	2018	~ 3.77	e^+e^-	0.8 %	BABAYAGA 3.5	Chin Phys C42 (2018) 063
BESIII	2022	4.0 – 4.6	e^+e^-	0.66 %	BABAYAGA@NLO	Chin Phys C46 (2022) 113
BESIII	2022	4.61 – 4.95	e^+e^-	0.53 %	BABAYAGA@NLO	Chin Phys C46 (2022) 113
BESIII	2024	3.77	e^+e^-	0.4 %	BABAYAGA@NLO	Chin Phys C48 (2024) 123

Overview Selection of Bhabha-Events



1. Selection of two charged tracks with opposite charge originating from IP
2. Require both tracks to be in fiducial volume
   - Association of track to EMC cluster
3. Require **back-to-back signature**
   - beam momenta close to half of cms energy
   - accolinearity cuts (2dim. or 3dim.) removes also gamma-gamma events
4. Particle Identification Cuts
   - E/p: Energy release in EMC/mometum of track
5. Remove remaining background events
   - mu+mu-, pi+pi- (percent level)
   - gamma-gamma events

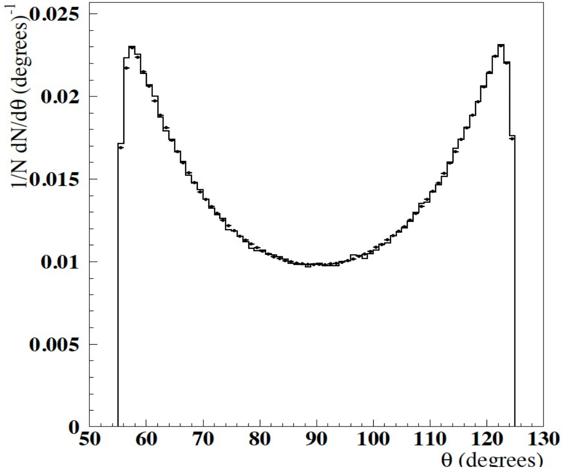
KLOE Luminosity Measurement $\sqrt{s}$ ~1 GeV

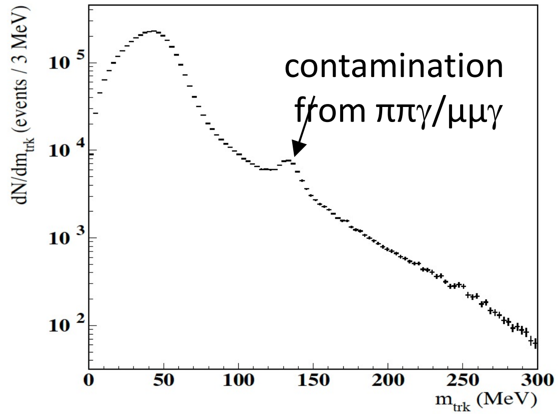
Features (uncertainty 0.6%):

- Preselection with calorimetric information only
- Refined selection including drift chamber information
  $55° < \Theta_{track} < 125°$
- Track momenta $p > 400\ MeV$
- Accolinearity $\zeta = |\theta_{cl1} + \theta_{cl2} - 180°| < 9°$

	correction (%)	systematic error (%)
angular acceptance	+0.25	0.25
tracking	–	0.06
clustering	+0.14	0.11
background	-0.62	0.13
cosmic veto	+0.40	–
energy calibration	–	0.10
center of mass energy	+0.10	0.10
	+0.34	0.32

+ BABAYAGA uncertainy 0.5%

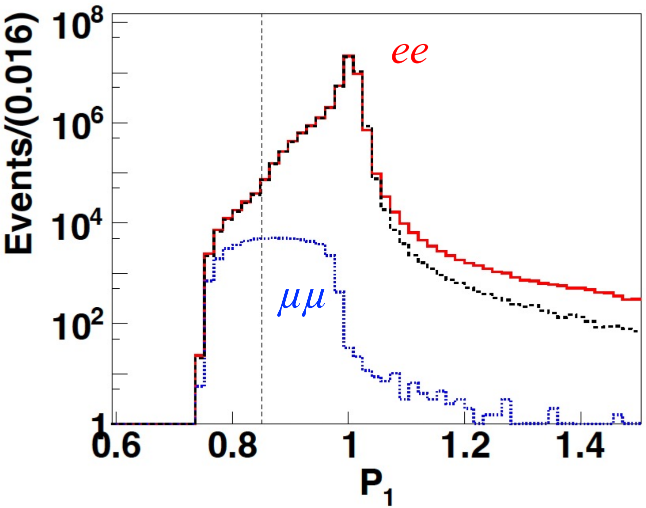




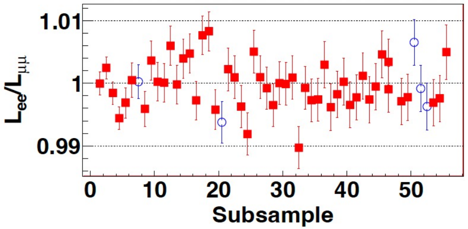
BABAR Luminosity Measurement $\sqrt{s}$ *~10.6 GeV*

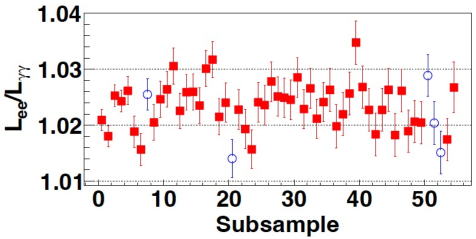
Features (uncertainty 0.43% for e^+e^-, 0.88% for $\mu^+\mu^-$):

- Downscaling of Bhabha events at small polar angles
- Separation of electrons/muons by means of E/p
- Bhabha selection (similar for $\mu^+\mu^-$)
  $|\cos(\Theta_{\text{track},1})| < 0.7$ rad, $|\cos(\Theta_{\text{track},1})| < 0.65$ rad
  $p_{\text{track},1} > 75\% \cdot \sqrt{s}/2$, $p_{\text{track},2} > 50\% \cdot \sqrt{s}/2$
  Accolinearity $\alpha < 30°$
  E/p > 0.7 (0.4)
- Additional cross check with $\gamma\gamma$ events

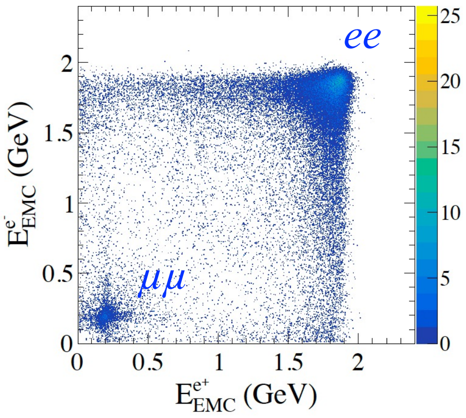


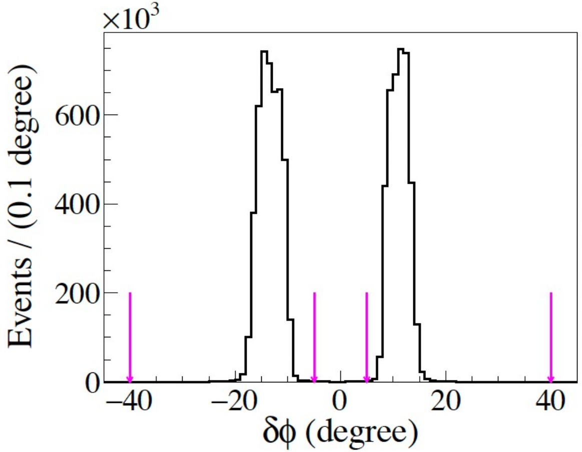
Source	Relative uncertainty on $\mathcal{L}$ (%)
Theoretical cross section	0.26 (e^+e^-), 0.44 ($\mu^+\mu^-$)
Track-reconstruction efficiency	0.13 (Runs 1–6), 0.20 (Run 7)
Trigger & offline-filter efficiency	0.10
Data-MC differences	0.5–0.7 (e^+e^-), 0.24–0.28 ($\mu^+\mu^-$)
Time dependence	0.16–0.46 (Off-resonance)
Background subtraction	0.02 (Runs 1–6), 0.10 ($\Upsilon(3S)$), 0.15 ($\Upsilon(2S)$)
Boost uncertainty	0.2 (Run 7)





BESIII Luminosity Measurement $\sqrt{s}$ ~3.77 GeV





Features (uncertainty 0.4%):

- Bhabha selection
  $|\cos(\Theta_{track,1})| < 0.8$ rad (barrel EMC)
  $p_{track,1} + p_{track,2} > 90\% \cdot \sqrt{s}$
  $1.0\text{ GeV} < E_{EMC} < 2.5\text{ GeV}$
  Accolinearity in polar angle $5° < |\delta\phi| < 40°$
- Requires strict cuts in momentum due to background from ISR production of J/ψ

Source	Uncertainty (%)
MDC information	0.29
E_{EMC}	0.16
$\cos\theta$	0.13
$\delta\phi$	0.02
Trigger	0.10
Generator	0.10
Total	0.38

Overview Selection Cuts

Main control channel used for luminosity measurement: Large-angle Bhabha events

- high statistics → optimal signal-to-background ratio
- full energy deposit in EMC → clear signature
- allows to combine tracking and EMC information → complementary information

Similar selection used by all modern collider experiments:

Experiment	$\sqrt{s}$ / GeV	Polar Angle Tracks	Acceptance Momentum p_{min} / $\sqrt{s}/2$	Accolinearity cut
KLOE	~ 1	55° - 125°	80%	9° polar angle
BABAR	~10	45° - 135°	75% (50%)	30° 3dim. angle
BESIII	~ 4	37° - 143°	90%	>5°, <40° azimuthal angle

Conclusions

Conclusions

- **Luminosity measurements** mainly based on **Bhabha reference channel**
  → cross checks with di-muon and $\gamma\gamma$ – channel
- Selection mainly based on Born process and (relatively) soft radiation
  → **BABAYAGA@NLO** provides already very high accuracy ~0.1%
- **Experimental issues in luminosity measurement beyond event generator currently limiting the overall accuracy**
  → trigger, background, electronics, hidden inefficiencies, saturation, ...
- **Bhabha event generator with Born and hard radiation needed for many applications in precision physics – presently not existing**
  → Background processes, multi-photon background, ...

Thank you !