Luminosity overview: theory

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Reference processes for luminosity

- Instead of getting the luminosity from machine parameters, it's more effective to exploit the relation

$$\sigma = \frac{N}{L} \quad \rightarrow \quad L = \frac{N_{\rm ref}}{\sigma_{\rm theory}} \qquad \frac{\delta L}{L} = \frac{\delta N_{\rm ref}}{N_{\rm ref}} \oplus \frac{\delta \sigma_{\rm theory}}{\sigma_{\rm theory}}$$

- Reference (*normalization*) processes are required to have a clean topology, high statistics and be calculable with high theoretical accuracy

⋆ **Large-angle** QED processes as $e^+e^- \to e^+e^-$ (Bhabha), $e^+e^- \to \gamma\gamma$, $e^+e^- \to \mu^+\mu^-$ are golden processes at flavour factories to achieve a typical precision at the level of $1 \div 0.1\%$

$\hookrightarrow$ QED radiative corrections are mandatory

⋆ At LEP and future high-energy e^+e^- machines **small-angle** Bhabha scattering is the golden process

A recent example: $\mathbf{N}_\nu$ from $\mathbf{\Gamma}_Z^{\mathrm{inv}}$ at LEP $\mathbf{Z}$ peak measurements

- assuming lepton universality

$$N_\nu \left(\frac{\Gamma_{\nu\bar{\nu}}}{\Gamma_{ll}}\right)_{\mathrm{SM}} = \sqrt{\frac{12\pi R_l^0}{\sigma^0_{\mathrm{had}} m_Z^2}} - R_l^0 - (3+\delta_\tau)$$

$$\mathbf{N}_\nu = \mathbf{2.9840 \pm 0.0082}$$

$$\delta N_\nu \simeq 10.5\frac{\delta n_{\mathrm{had}}}{n_{\mathrm{had}}} \oplus 3.0\frac{\delta n_{\mathrm{lept}}}{n_{\mathrm{lept}}} \oplus 7.5\frac{\delta \mathcal{L}}{\mathcal{L}}$$

$$\frac{\delta \mathcal{L}}{\mathcal{L}} = 0.061\% \Longrightarrow \delta N_\nu = 0.0046$$

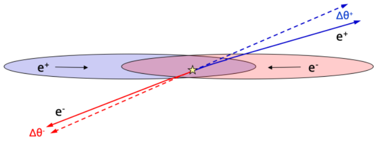
ADLO, SLD and LEPEWWG, Phys. Rept. 427 (2006) 257, hep-ex/0509008

2σ away from SM: hint for BSM? Right handed neutrinos?

Beam-beam effects studied in detail recently

G. Voutsinas, E. Perez, M. Dam, P. Janot, arXiv:1908.01704

- systematics bias on the acceptance due to e.m. beam-beam interactions $\Longrightarrow$ underestimate of luminosity by $\sim 0.1\%$



- together with an update on Bhabha cross sections (see later) $\Longrightarrow$ Luminosity

P. Janot, S. Jadach, arXiv:1912.02067

$$\mathbf{N_\nu = 2.9963 \pm 0.0074}$$

Luminosity is a key quantity for a precision e^+e^- collider

Revisiting luminosity th. err. at LEP & prospects for future machines

Past and recent updates

- **theoretical error in SABS at LEP1 by the end of operation**

Type of correction/error	(%)	(%)	updated (%)
missing photonic $O(\alpha^2 L)$	0.100	0.027	0.027
missing photonic $O(\alpha^3 L^3)$	0.015	0.015	0.015
vacuum polarization	0.040	0.040	0.040
light pairs	0.030	0.030	0.010
Z-exchange	0.015	0.015	0.015
total	0.110	0.061	0.054

I column: S. Jadach, O. Nicrosini et al. Physics at LEP2 YR 96-01, Vol. 2
A. Arbuzov et al., Phys. Lett. B389 (1996) 129
II column: B.F.L. Ward, S. Jadach, M. Melles, S.A. Yost, hep-ph/9811245
III column: G. Montagna et al., Nucl. Phys. B547 (1999) 39

- **experimental systematics: 0.034%**

G. Abbiendi et al., (OPAL), Eur. Phys. J. C14 (2000) 373

- **recent reanalysis**
  - "The path to 0.01% theoretical luminosity precision for the FCC-ee"

    S. Jadach, W. Placzek, M. Skrzypek, B.F.L. Ward and S.A. Yost, Phys Lett B790 (2019) 314
  - "Improved Bhabha cross section at LEP and the number of light neutrino species"

    P. Janot and S. Jadach, Phys. Lett. B803 (2020) 135319
  - "Study of theoretical luminosity precision for electron colliders at higher energies"

    S. Jadach, W. Placzek, M. Skrzypek and B.F.L. Ward, Eur. Phys. J. C81 (2021) 1047

Type of correction / Error	Update 2018	FCC-ee forecast
(a) Photonic $[\mathcal{O}(L_e\alpha^2)]$ $\mathcal{O}(L_e^2\alpha^3)$	0.027%	0.1×10^{-4}
(b) Photonic $[\mathcal{O}(L_e^3\alpha^3)]$ $\mathcal{O}(L_e^4\alpha^4)$	0.015%	0.6×10^{-5}
(c) Vacuum polariz.	0.014% [26]	0.6×10^{-4}
(d) Light pairs	0.010% [18, 19]	0.5×10^{-4}
(e) Z and s-channel γ exchange	0.090% [11]	0.1×10^{-4}
(f) Up-down interference	0.009% [28]	0.1×10^{-4}
(f) Technical Precision	(0.027)%	0.1×10^{-4}
Total	0.097%	1.0×10^{-4}

S. Jadach, W. Placzek, M. Skrzypek, B.F.L. Ward and S.A. Yost, Phys Lett B790 (2019) 314

Forecast study for FCCee_{M_Z}

Type of correction / Error	Published [2]	Redone
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.10×10^{-4}	0.10×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.06×10^{-4}	0.06×10^{-4}
(b') Photonic $\mathcal{O}(\alpha^2 L_e^0)$		0.17×10^{-4}
(c) Vacuum polariz.	0.6×10^{-4}	0.6×10^{-4}
(d) Light pairs	0.5×10^{-4}	0.27×10^{-4}
(e) Z and s-channel γ exch.	0.1×10^{-4}	0.1×10^{-4}
(f) Up-down interference	0.1×10^{-4}	0.08×10^{-4}
Total	1.0×10^{-4}	0.70×10^{-4}

Forecast

Type of correction / Error	ILC_{500}	ILC_{1000}	CLIC_{3000}
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.13×10^{-4}	0.15×10^{-4}	0.20×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.27×10^{-4}	0.37×10^{-4}	0.63×10^{-4}
(c) Vacuum polariz.	1.1×10^{-4}	1.1×10^{-4}	1.2×10^{-4}
(d) Light pairs	0.4×10^{-4}	0.5×10^{-4}	0.7×10^{-4}
(e) Z and s-channel γ exch.	$1.0 \times 10^{-4(*)}$	2.4×10^{-4}	16×10^{-4}
(f) Up-down interference	$< 0.1 \times 10^{-4}$	$< 0.1 \times 10^{-4}$	0.1×10^{-4}
Total	1.6×10^{-4}	2.7×10^{-4}	16×10^{-4}

B.F.L. Ward, S. Jadach, W. Placzek, M. Skrzypek, S.A. Yost, arXiv:2410.09095

Summary of QED (photonic) radiative corrections

Loosely and schematically, the corrections to the LO cross section can be arranged as ($L \equiv \log \frac{s}{m_e^2}$ = collinear log)

LO	α^0		
NLO	αL	α	
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
h.o.	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$	$\dots$

Blue: Leading-Log PS, Leading-Log YFS, SF

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Red: PS matched to NLO, YFS, SF + NLO

Loosely and schematically, the corrections to the LO cross section can be arranged as ($L \equiv \log \frac{s}{m_e^2}$ = collinear log)

LO	90%		
NLO	10%	0.5%	
NNLO	0.5%	0.05%	0.01%
h.o.	0.01%	$\cdots$	$\cdots$

Tipically at flavour factories (on integrated Bhabha σ)

Possible additional enhancements from IR logs induced by events selection

Matching NLO and a QED Parton Shower in **BabaYaga@NLO**

Balossini et al., Phys. Lett. **663** (2008) 209; Balossini et al., Nucl. Phys. **B758** (2006) 227

C.M.C.C., Phys. Lett. B **520** (2001) 16; C.M.C.C. et al., Nucl. Phys. B **584** (2000) 459

- **BabaYaga@NLO** is based on a QED Parton Shower matched with exact NLO matrix elements, such that:

⇝ $\left[\sigma^{\infty}_{matched}\right]_{\mathcal{O}(\alpha)} = \sigma^{\alpha}_{\mathrm{NLO}}$

⇝ resummation of higher orders LL (PS) contributions is preserved

⇝ the cross section is fully differential in the momenta of the final state particles (e^+, e^- and $n\gamma$)

⇝ as a by-product, part of photonic $\alpha^2 L$ included by means of the matching procedure

G. Montagna et al., **PLB** 385 (1996)

⇝ the theoretical error starts with $\mathcal{O}(\alpha^2)$ (NNLO) not infrared, singly collinear terms: very naively and roughly (for photonic corrections)

$$\frac{1}{2}\alpha^2 L \equiv \frac{1}{2}\alpha^2 \log\frac{s}{m_e^2} \sim 5\times 10^{-4}$$

Results with **BabaYaga@NLO** for luminometry

- to show the typical size of RC, the following setups and definitions are used (for Bhabha)

  a. $\sqrt{s} = 1.02\ \mathrm{GeV},\ E_{min} = 0.408\ \mathrm{GeV},\ 20^\circ < \theta_\pm < 160^\circ,\ \xi_{max} = 10^\circ$
  b. $\sqrt{s} = 1.02\ \mathrm{GeV},\ E_{min} = 0.408\ \mathrm{GeV},\ 55^\circ < \theta_\pm < 125^\circ,\ \xi_{max} = 10^\circ$
  c. $\sqrt{s} = 10\ \mathrm{GeV},\ E_{min} = 4\ \mathrm{GeV},\ 20^\circ < \theta_\pm < 160^\circ,\ \xi_{max} = 10^\circ$
  d. $\sqrt{s} = 10\ \mathrm{GeV},\ E_{min} = 4\ \mathrm{GeV},\ 55^\circ < \theta_\pm < 125^\circ,\ \xi_{max} = 10^\circ$

$$\delta_{VP} \equiv \frac{\sigma_{0,VP} - \sigma_0}{\sigma_0} \qquad \delta_\alpha \equiv \frac{\sigma_\alpha^{NLO} - \sigma_0}{\sigma_0}$$

$$\delta_{HO} \equiv \frac{\sigma_{matched}^{PS} - \sigma_\alpha^{NLO}}{\sigma_0} \qquad \delta_{HO}^{PS} \equiv \frac{\sigma^{PS} - \sigma_\alpha^{PS}}{\sigma_0}$$

$$\delta_\alpha^{non\text{-}log} \equiv \frac{\sigma_\alpha^{NLO} - \sigma_\alpha^{PS}}{\sigma_0} \qquad \delta_\infty^{non\text{-}log} \equiv \frac{\sigma_{matched}^{PS} - \sigma^{PS}}{\sigma_0}$$

Results with **BabaYaga@NLO** for luminometry

setup	(a)	(b)	(c)	(d)
δ_{VP}	1.76	2.49	4.81	6.41
δ_{α}	-11.61	-14.72	-16.03	-19.57
δ_{HO}	0.39	0.82	0.73	1.44
δ_{HO}^{PS}	0.35	0.74	0.68	1.34
$\delta_{\alpha}^{non\text{-}log}$	-0.34	-0.56	-0.34	-0.56
$\delta_{\infty}^{non\text{-}log}$	-0.30	-0.49	-0.29	-0.46

Table: Relative corrections (in per cent) to the Bhabha cross section for the four setups

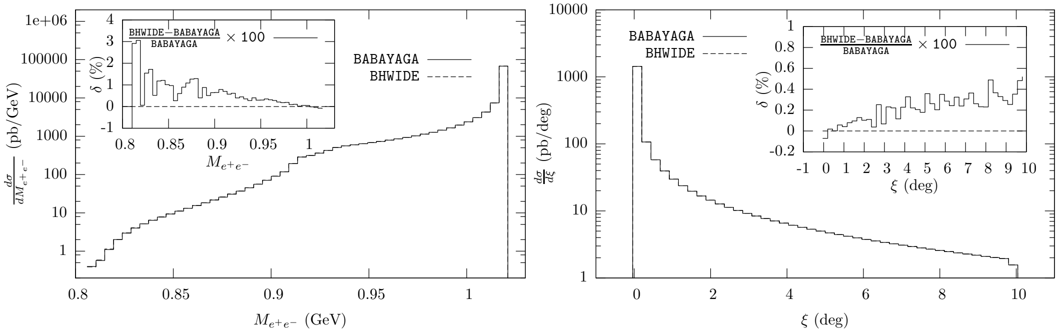
- ⋆ in short, the fact that $\delta_{\alpha}^{non\text{-}log} \simeq \delta_{\infty}^{non\text{-}log}$ and $\delta_{HO} \simeq \delta_{HO}^{PS}$ means that the matchin algorithm preserves both the advantages of exact NLO calculation and PS approach:
  - ↦ it includes the missing NLO RC to the PS
  - ↦ it adds the missing higher-order RC to the NLO

Estimating the theoretical accuracy

in “Luminosity”, S. Actis et al., Eur Phys. J. **C 66** (2010), 585

“Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data

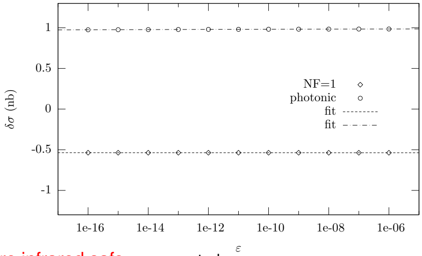
- A sound estimate of the theoretical accuracy & error must go through a careful comparison of independent calculations/codes, in order to
  - ⇝ assess the technical precision and spot bugs (with the same theory)
  - ⇝ estimate the theoretical error when including partial/incomplete higher-order corrections

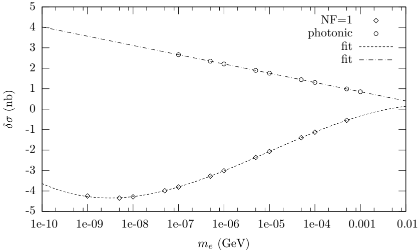


Comparison of $\sigma_{\rm SV}^{\alpha^2}$ calculation of **`BabaYaga@NLO`** with

Using realistic cuts for luminosity @ KLOE

- A. Penin (PRL **95** (2005) 010408 & Nucl. Phys. **B734** (2006) 185, i.e. NNLO photonic corrections): as a function of the log of the soft photon cut-off and of a fictitious electron mass





★ differences are infrared safe, as expected

★ $\delta\sigma(\text{photonic})/\sigma_0 \propto \alpha^2 L$, as expected

- Numerically, for various selection criteria at the Φ and B factories

$$\sigma_{\rm SV}^{\alpha^2}(\text{Penin}) - \sigma_{\rm SV}^{\alpha^2}(\texttt{BabaYaga@NLO}) < 0.02\% \times \sigma_0$$

Lepton and hadron loops & pairs at NNLO

from an old talk (2014)

- The exact NNLO soft+virtual corrections and $2 \to 4$ matrix elements $e^+e^- \to e^+e^-(l^+l^-, l = e, \mu, \tau), e^+e^-(\pi^+\pi^-)$ are now available
- In comparison with the *approximation* in BabaYaga@NLO and using realistic luminosity cuts ($S_i \equiv \sigma_i^{\text{NNLO}}/\sigma_{BY}$)

	$\sqrt{s}$		σ_{BY}	$S_{e^+e^-}$ [‰]	S_{lep} [‰]	S_{had} [‰]	S_{tot} [‰]
KLOE	1.020	NNLO		-3.935(4)	-4.472(4)	1.02(2)	-3.45(2)
		BB@NLO	455.71	-3.445(2)	-4.001(2)	0.876(5)	-3.126(5)
BES	3.650	NNLO		-1.469(9)	-1.913(9)	-1.3(1)	-3.2(1)
		BB@NLO	116.41	-1.521(4)	-1.971(4)	-1.071(4)	-3.042(5)
BaBar	10.56	NNLO		-1.48(2)	-2.17(2)	-1.69(8)	-3.86(8)
		BB@NLO	5.195	-1.40(1)	-2.09(1)	-1.49(1)	-3.58(2)
Belle	10.58	NNLO		-4.93(2)	-6.84(2)	-4.1(1)	-10.9(1)
		BB@NLO	5.501	-4.42(1)	-6.38(1)	-3.86(1)	-10.24(2)

★ The uncertainty due to lepton and hadron pair corrections is at the level of a few units in 10^{-4}

Carloni, Czyz, Gluza, Gunia, Montagna, Nicrosini, Piccinini, Riemann *et al.*, JHEP **1107** (2011) 126 from the same old talk

Status of the MC theoretical accuracy

Main conclusion of the Luminosity Section of the WG Report

Putting the various sources of uncertainties (for large–angle Bhabha) all together...

Source of error (%)	$\Phi-$factories	$\sqrt{s}$ = 3.5 GeV	$B-$factories
$\lvert\delta^{\rm err}_{\rm VP}\rvert$ [Jegerlehner]	0.00	0.01	0.03
$\lvert\delta^{\rm err}_{\rm VP}\rvert$ [HMNT]	0.02	0.01	0.02
$\lvert\delta^{\rm err}_{{\rm SV},\alpha^2}\rvert$	0.02	0.02	0.02
$\lvert\delta^{\rm err}_{{\rm HH},\alpha^2}\rvert$	0.00	0.00	0.00
$\lvert\delta^{\rm err}_{{\rm SV,H},\alpha^2}\rvert$ [in progress]	0.05	0.05	0.05
$\lvert\delta^{\rm err}_{\rm pairs}\rvert$	0.03	0.016	0.03
$\lvert\delta^{\rm err}_{\rm total}\rvert$ linearly	0.12	0.1	0.13
$\lvert\delta^{\rm err}_{\rm total}\rvert$ in quadrature	0.07	0.06	0.06

- For the experiments on top of and closely around the J/ψ resonance, the accuracy slightly deteriorates, because of the differences between the predictions of independent $\Delta\alpha^{(5)}_{\rm had}(q^2)$ routines

★ The present error estimate appears to be rather robust and sufficient for high–precision luminosity measurements. It is comparable with that achieved for small–angle Bhabha luminosity monitoring at LEP/SLC

- Also $e^+e^- \to \gamma\gamma$ can be used as reference process for luminometry:

⇝ even if it has lower cross-section and larger background

⇝ from the theoretical point of view: "only" ISR (smaller RCs), VP enters only at NNLO

⇝ It's implemented in **`BabaYaga@NLO`**, in the same framework: Parton Shower + NLO

$\sqrt{s}$ (GeV)	1	3	10
σ	137.53	15.281	1.3753
$\sigma_\alpha^{\mathrm{PS}}$	128.55	14.111	1.2529
$\sigma_\alpha^{\mathrm{NLO}}$	129.45	14.211	1.2620
$\sigma_{\mathrm{exp}}^{\mathrm{PS}}$	128.92	14.169	1.2597
σ_{exp}	129.77	14.263	1.2685
δ_α	-5.87	-7.00	-8.24
δ_∞	-5.65	-6.66	-7.77
δ_{exp}	0.24	0.37	0.51
$\delta_\alpha^{\mathrm{NLL}}$	0.70	0.71	0.73
$\delta_\infty^{\mathrm{NLL}}$	0.66	0.66	0.69

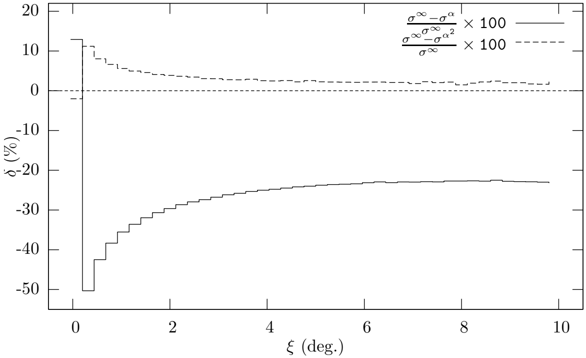
Table: Photon pair production cross sections (in nb) to different accuracy levels and relative corrections (in per cent)

Conclusions

- Precise luminosity knowledge is important for key measurements at flavour factories
- Monte Carlo generators are essential tools to account for experimental cuts & interface to detector simulation
- Theoretical accuracy must be firmly assessed
- Now Bhabha is implemented at NNLO in **`McMule`** P. Banerjee et al., PLB 820 (2021) 136547
- $e^+e^- \to \gamma\gamma$ at NNLO next talk by Marco
- Is it time to re-visit and re-asses the theoretical accuracy for luminosity at low-energy e^+e^- machines, under the light of new developments and independent calculations?

SPARES

- With a complete NNLO generator at hand, can LL resummation beyond α^2 be neglected (Bhabha at KLOE)?



ξ = acollinearity

- Resummation beyond α^2 still important (at least for some distributions)!