UNIVERSITÀ DI TORINO McMULE

Workshop on Radiative Corrections and Monte Carlo Simulations for e^+e^- collisions

$e^+e^- \to \gamma\gamma$ at NNLO

Marco Rocco

for the McMule Team

Università di Torino & INFN

Pisa, May 5$^{\text{th}}$, 2025

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$e^+e^- \to \gamma\gamma$ at NNLO with McMule

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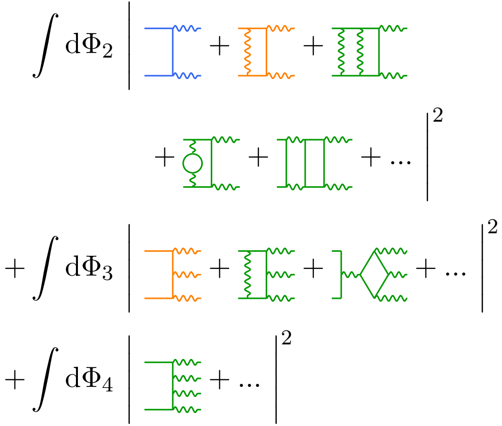
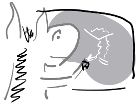
- luminosity in general has been discussed
- the role of $ee \to \gamma\gamma$ is clear
- as is the role of QED corrections
- this is the environment where McMULE thrives
- now fully differential NNLO QED corrections, including electron-mass and fermion-loop effects

code $\to$ `https://gitlab.com/mule-tools/mcmule/-/tree/digamma?ref_type=heads`

docs $\to$ `https://mcmule.readthedocs.io/`

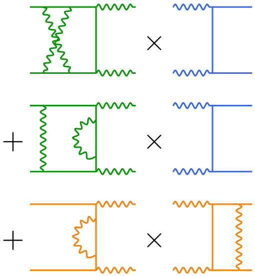


1. fully-differential PS integration
   $\to$ FKS$^\ell$ and PS partitioning
2. numerical instabilities due to pseudo-singularities
3. virtual amplitudes with massive particles



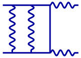
a peek in the ~~stable~~ theory

two-loop diagrams *without* fermion loops



$+\ldots$

- use $m_e^2 \ll Q^2 \rightarrow$ expand in m_e^2/Q^2



$$\sim A \log^2 \frac{m_\ell^2}{Q^2} + B \log \frac{m_\ell^2}{Q^2} + C + \mathcal{O}\Big(\frac{m_\ell^2}{Q^2}\Big)$$

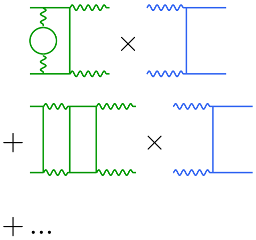
- apply massification factor Z for each massified leg ($\mathcal{S} = 1$ here):

$$\mathcal{A}(m_e) = \mathcal{S} \times Z \times Z \times \mathcal{A}(0) + \mathcal{O}(m_e)$$

[Penin 06, Becher, Melnikov 07; Engel, Gnendiger, Signer, Ulrich 18, Ulrich 23]

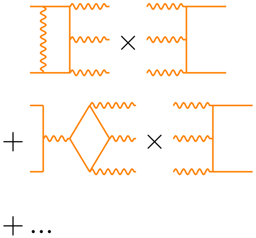
- use [Anastasiou, Glover, Tejeda-Yeomans 02] for $\mathcal{A}(0)$

two-loop diagrams *with* fermion loops



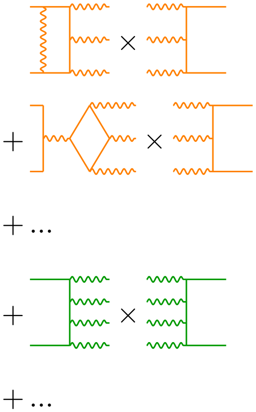
- Vacuum Polarisation diagrams (◯)
  - hyperspherical method [Fael 18]
  - dispersive method
  - no particular reason for preferring one over the other
- Light-By-Light diagrams (□)
  - use [Anastasiou, Glover, Tejeda-Yeomans 02] for $\mathcal{A}(0)$ with massless electron
  - see later for phenomenological contribution
- massification is trickier for VP ($\mathcal{S} \neq 1$), while $\mathcal{S} = Z = 1$ for LBL

one-loop diagrams at NNLO



- OpenLoops [Buccioni, Pozzorini, Zoller 18, Buccioni et al. 19], for both bosonic and fermionic (LBL, electronic only) loops
- stability in soft and pseudo-collinear phase space regions
  - no need for next-to-soft stabilisation [LBK 58-61, McMule 21, Engel, Signer, Ulrich 21, 2xEngel 23]
  - crossing to Compton scattering might require it

one-loop and tree-level diagrams at NNLO



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  - crossing to Compton scattering might require it
- optimised analytic implementation of lengthy double-real expression
- phase-space tuning and partitioning improve convergence esp. at higher energies

phenomenology at low energies (1–10 GeV)

comparison with RMCL1 i.e. [0912.0749]

$$e^+e^- \to \gamma\gamma \ @ \ \{1,3,10\}\,\mathrm{GeV}$$

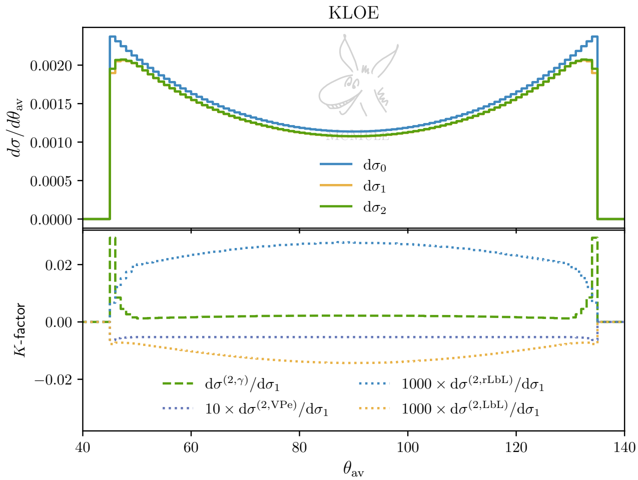
> at least two accepted photons (l and s are the most energetic) with

$$E_\gamma \geq 0.3\sqrt{s}\,, \quad 45^\circ \leq \theta_\gamma \leq 135^\circ\,, \quad \xi \equiv |\theta_l + \theta_s - \pi| \leq 10^\circ$$

$\sqrt{s}$	1 GeV		3 GeV		10 GeV	
	McMule [nb]	RMCL1 [nb]	McMule [nb]	RMCL1 [nb]	McMule [nb]	RMCL1 [nb]
σ_0	137.531	137.53	15.2812	15.281	1.375	1.3753
σ_1	129.444	129.45	14.2099	14.211	1.26185	1.2620
σ_2	129.760		14.2570		1.26738	
$\sigma_{1+\mathrm{PS}}$		129.77		14.263		1.2685

$e^+e^- \to \gamma\gamma$ @ $\{1, 3, 10\}$ GeV

$\sqrt{s}$	1 GeV		3 GeV		10 GeV	
	McMule [nb]	$\lvert\delta K\rvert$	McMule [nb]	$\lvert\delta K\rvert$	McMule [nb]	$\lvert\delta K\rvert$
σ_0	137.531		15.2812		1.375	
σ_1	129.444	5.9%	14.2099	7.0%	1.26185	8.2%
σ_2	129.760	0.2%	14.2570	0.3%	1.26738	0.4%
$\sigma^{(2,\gamma)}$	0.383	0.3%	0.0598	0.4%	0.00738	0.6%
$\sigma^{(2,\text{VP}e)}$	− 0.069	0.1%	− 0.0128	0.1%	− 0.00186	0.2%
$\sigma^{(2,\text{LbL}e)}$	− 0.0014		− 0.00016		− 0.000014	
$\sigma^{(2,\text{rLbL}e)}$	0.0030		0.00033		0.000030	
$\sigma^{(2,\text{VP}\mu\tau)}$	− 0.000077		− 0.00018		− 0.000080	
$\sigma^{(2,\text{VPh})}$	0.000090		− 0.00010		− 0.000097	



$$\theta_{\rm av} = \frac{1}{2}(\theta_l - \theta_s + \pi)$$

$e^- e^+ \to \gamma\gamma$ @ $\sqrt{s} = 1$ GeV

> at least two photons satisfying

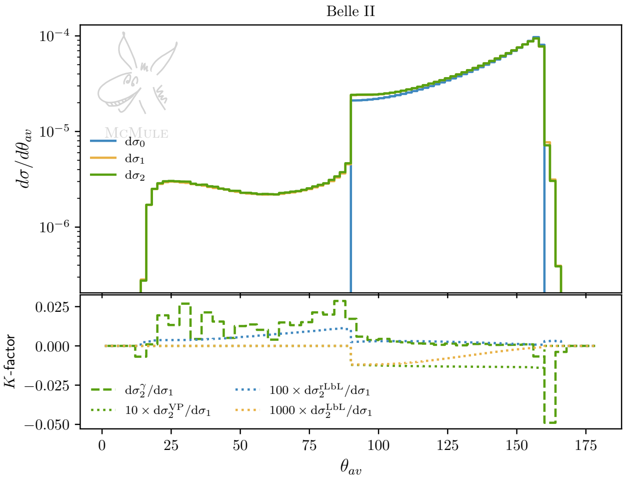
$$E_\gamma \geq 300\,\mathrm{MeV}$$

$$45^\circ \leq \theta_\gamma \leq 135^\circ$$

$$\xi \equiv |\theta_l + \theta_s - \pi| \leq 10^\circ$$

> Gaussian beam spread of FWHM of 0.12 MeV [KLOE 2005] is applied

→ cures soft enhancement when energies of photons are equal



$$\theta_{\mathrm{av}} = \frac{1}{2}(\theta_l - \theta_s + \pi)$$

$$e^-(7\,\mathrm{GeV})\,e^+(4\,\mathrm{GeV}) \to \gamma\,\gamma$$

@ $\sqrt{s} = 10.583$ GeV

> at least two photons satisfying

$$E_\gamma \geq 1\,\mathrm{GeV}$$

$$15^\circ \leq \theta_\gamma \leq 165^\circ$$

> no beam spread is applied

a mule never stops

- McMule provides NNLO QED corrections for $e^+e^- \to \gamma\gamma$ with electron-mass and fermion-loop effects throughout (most of) the calculation
- we agree with previous results for low-energy phenomenology
- crossing to Compton scattering ($e\gamma \to e\gamma$) is being put in place

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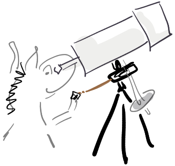
future steps of the mule



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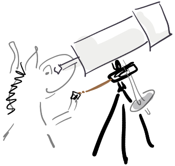
- compare with other RMCL2 codes



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- compare with other RMCL2 codes
- improve stability at higher energies

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- compare with other RMCL2 codes
- improve stability at higher energies
- include electroweak effects

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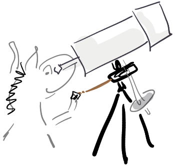
future steps of the mule

◇ compare with other RMCL2 codes
◇ improve stability at higher energies
◇ include electroweak effects
◇ LBL for any fermion loop



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future steps of the mule

- ◇ compare with other RMCL2 codes
- ◇ improve stability at higher energies
- ◇ include electroweak effects
- ◇ LBL for any fermion loop
- ◇ world dominance

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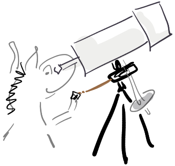
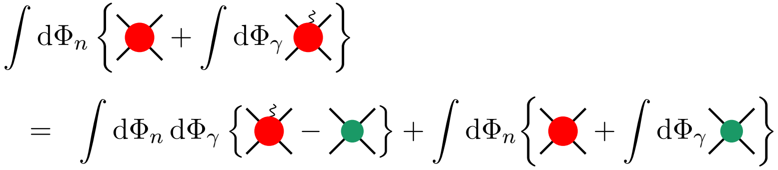


Thank you!

a universal recipe for divergent radiative corrections



- exploits exponentiation of soft singularities [YFS 61]
- works at all orders in QED [Engel, Signer, Ulrich 19]
- singularities are dealt with locally $\rightarrow$ stable numerical integration
- no resolution parameters $\rightarrow$ theory error: 0