Workshop on Radiative Corrections and Monte Carlo simulations for electron-positron collisions

Disperon QED
– Methods for internal and external hadrons –

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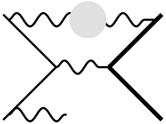
collaboration with Y. Fang, M. Rocco, A. Signer, Y. Ulrich, M. Zoller

what this talk is about

- **challenge** :: (data $\rightarrow$ loop integral) $\rightarrow$ phenomenology
- where
  - **internal hadrons** :: HVP (well known ...)
  - **external hadrons** :: pion poles F_π [Colangelo et al 22]
- how
  - (data $\rightarrow$ loop integral) :: disperson relations ✓
  - (data $\rightarrow$ loop integral) $\rightarrow$ pheno
    goal automatised and efficient as much as possible
- solution :: **Disperon QED**
  = **OpenLoops*** $\oplus$ **Disperon EFT** $\oplus$ **universal threshold subtraction**

*for McMule

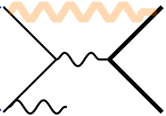
internal hadrons :: HVP



$\in ee \to \mu\mu\gamma$ @ NNLO $\quad \dfrac{\Pi(k^2)}{k^2} = -\dfrac{1}{\pi}\displaystyle\int_{4m_\pi^2}^{\infty} \mathrm{d}S \, \dfrac{\mathrm{Im}\Pi(S)}{S(k^2-S)}$

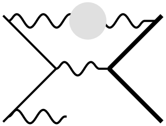
HVP treatment well established :: also hyperspherical method [Fael 18] (t-channel)

$$\Longrightarrow \int \mathrm{d}S \Bigg(\quad \Bigg)$$



. . . can be described by a massive dispersive photon, a disperon

internal hadrons :: HVP



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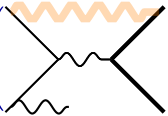
HVP treatment well established :: also hyperspherical method [Fael 18] (t-channel)

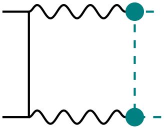
$$\Longrightarrow \int \mathrm{d}S \Big(\quad - \mathrm{CT} \Big) + \int \mathrm{d}S\, \mathrm{CT}$$



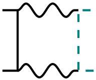
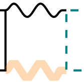
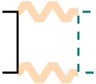
. . . can be described by a massive **dispers**ive phot**on**, a **disperon**

external hadrons :: pole contributions

example: $ee \to \pi\pi$ @ NLO FsQED [Colangelo et al 22]
(also calculated in [Budassi et al 24] and [Holz, Cottini])



$$\Longrightarrow \quad \frac{F_\pi(k^2)}{k^2} = \frac{1}{k^2} - \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \mathrm{d}S \, \frac{\mathrm{Im} F_\pi(S)}{S(k^2 - S)}$$

$\Longrightarrow$  $+ \int \mathrm{d}S \Big($  $\Big) + \int \mathrm{d}S_1 \mathrm{d}S_2$ 

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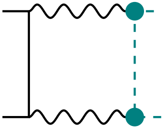
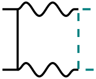
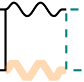
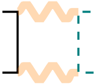


$$\Longrightarrow \frac{F_\pi(k^2)}{k^2} = \frac{1}{k^2} - \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \mathrm{d}S \, \frac{\mathrm{Im}F_\pi(S)}{S(k^2 - S)}$$

$\Longrightarrow$  $+ \int \mathrm{d}S \Big($  $- \mathrm{CT}\Big) + \int \mathrm{d}S \, \mathrm{CT} + \int \mathrm{d}S_1 \mathrm{d}S_2$ 

with threshold subtraction via counterterm for amplitude [McMule]
. . . can be described by a massive dispersive photon, a disperon

Disperon QED :: efficient automatisation of (data $\to$ loop integral) $\to$ pheno

- massive photon (disperon) with special Feynman rules
  $\Longrightarrow$ rely on **automated tools** :: McMule :: OpenLoops
- numerics :: dispersive integral $\in$ Monte Carlo integration
  $\Longrightarrow$ stable and fast evaluation, also for $S \to \infty$
  $\Longrightarrow$ switch from full matrix element to expanded version at $S = \Lambda \gg s, 4m_\pi^2$
  described by **Disperon EFT**

$$\int^{\Lambda} \mathrm{OpenLoops} + \int_{\Lambda}^{\infty} \mathrm{EFT}$$

- threshold singularities dealt with through an **universal** description

Disperon QED
$=$ **OpenLoops** $\oplus$ **Disperon EFT** $\oplus$ **universal threshold subtraction**

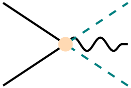
today: FsQED with Disperon QED

1. $ee \to \pi\pi$
   - Disperon EFT
   - RMCL2 results
2. extension to $ee \to \pi\pi\gamma$

EFT aspects

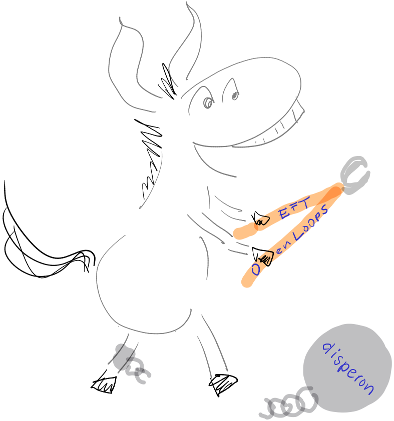
Why do we need an EFT?

EFT $\ni$ lin. independent* set of **gauge invariant** operators

- matching $ee \to \pi\pi$
  $\Longrightarrow$ operators with ∂_μ
- use gauge invariance $\partial_\mu \to D_\mu$
- get 
  $\Longrightarrow$ no need to redo matching

*not important here

EFT aspects



How do we pick Λ?

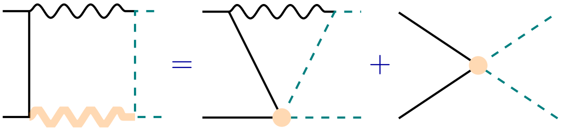
- similar to 'slicing'
- first guess based on

$$\text{OpenLoops}(S') \approx \text{EFT}(S')$$

  vary it and check obs. dependence
- $\int \frac{\mathrm{d}S}{S} \cdots \implies$ error can be large
- precision vs. speed
- $\Lambda = \Lambda(s)$

Pion Disperon EFT

disperons $S \geq \Lambda \gg s$ described by an 'EFT' up to dimension 8 # [MSc thesis Y. Fang]

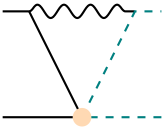


$$+ \mathcal{O}\left(\frac{1}{S^3}\right)$$

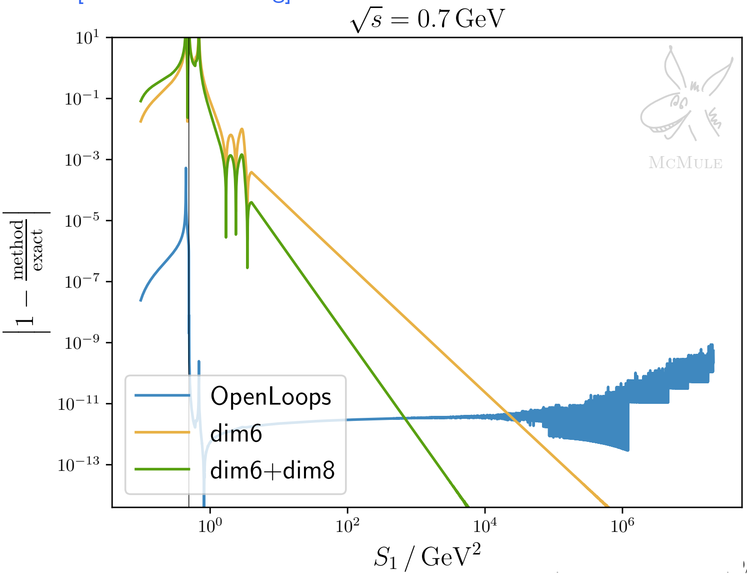
$$\mathcal{L}_{\gamma d} = \mathcal{L}_{\text{sQED}} + \sim \int_{\Lambda}^{\infty} \mathrm{d}S \frac{\text{Im} F_\pi(S)}{\pi S} \left\{ \frac{\cdots}{S} (\bar{\psi}\gamma^\mu\psi) \left[\pi^\dagger \left(i\overleftarrow{D}_\mu + i\overrightarrow{D}_\mu\right)\pi\right] + \cdots + \frac{\cdots}{S^2} \right\} + \mathcal{O}\left(\frac{1}{S^3}\right)$$

such that no field redefinitions required to obtain dim8 result

single-dispersive contributions [MSc thesis Y. Fang]

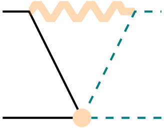


$\implies$ dim8 nice to have

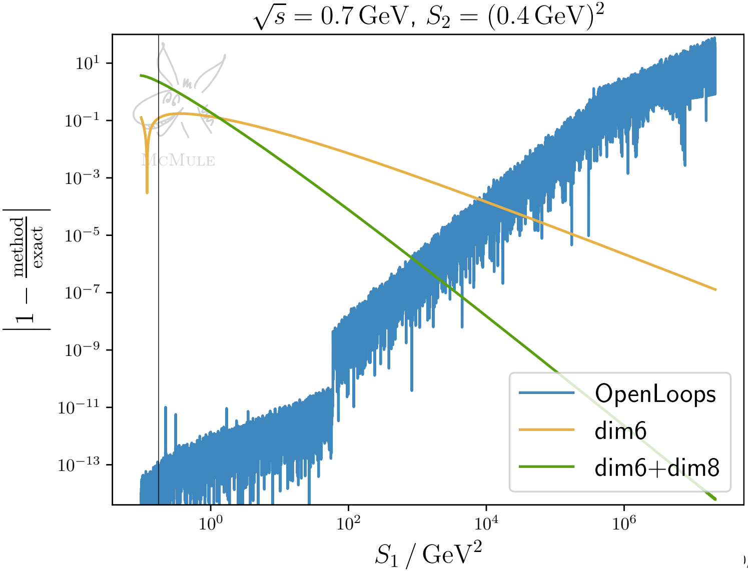


EFT performance vs. OpenLoops II

double-dispersive **single-heavy** contributions [MSc thesis Y. Fang]

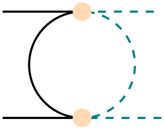


$\Longrightarrow$ dim8 nice to have



EFT performance vs. OpenLoops III

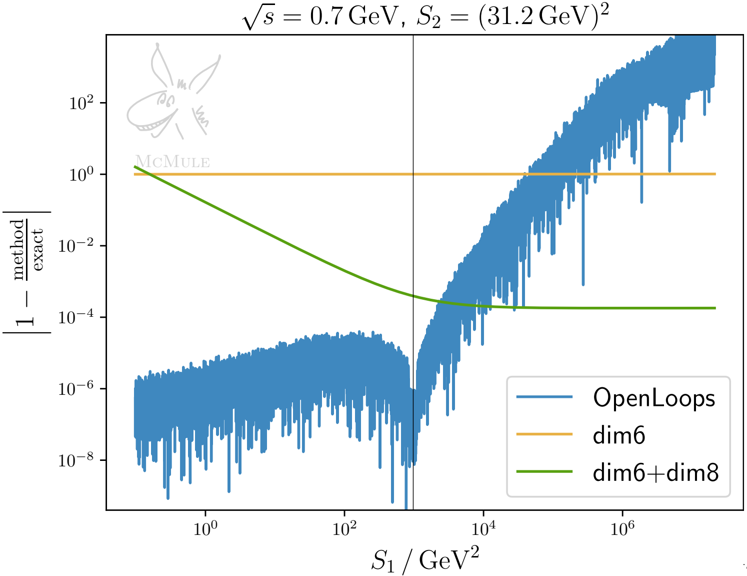
double-dispersive **double-heavy** contributions [MSc thesis Y. Fang]



$$\frac{t_{\text{OpenLoops}}}{t_{\text{EFT}}} \approx 100$$

$\Longrightarrow$ dim8 needed :: accidental cancellation in dim6 terms

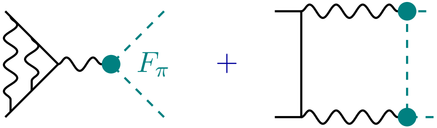
dim6 only $\sim m_e^2$ (seagull)



impact of FsQED :: McMule plot update

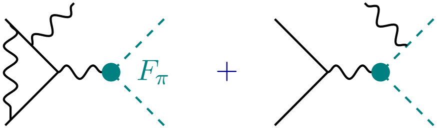
- since Liverpool 11/2024: updates* and test of FsQED implementation for $ee \to \pi\pi$

- $ee \to \pi\pi$ ::



example: CMD

- $ee \to \pi\pi\gamma$ ::

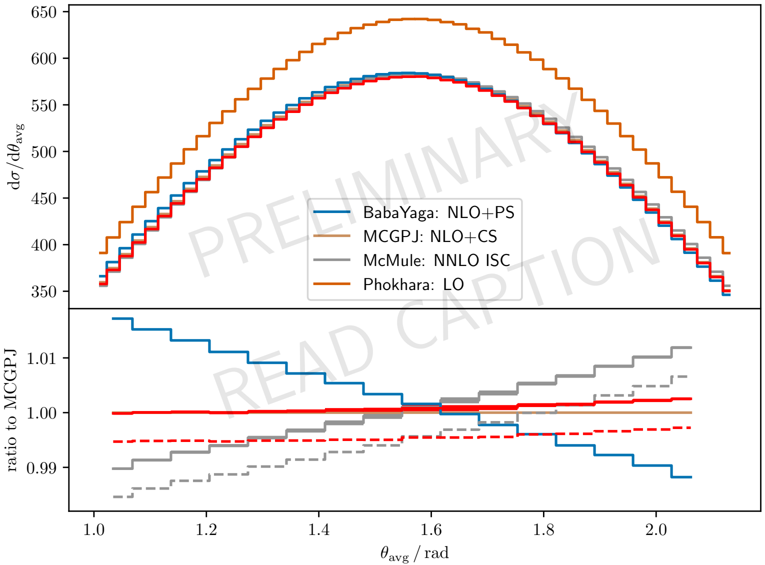


example: KLOE-LA

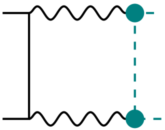
- public plots not yet updated, data + update plots available on gitlab

*uses a manual calculation, not yet OpenLoops

McMule plot update :: CMD $\sqrt{s} = 0.7\,\text{GeV}$



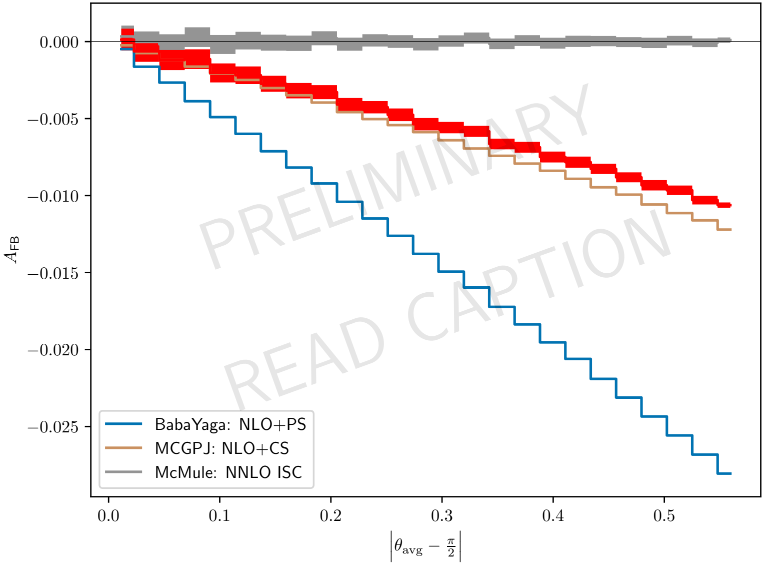
$\theta_{\text{avg}} =$
$(\theta^- - \theta^+ + \pi)/2$



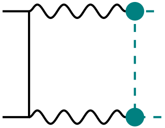
BabaYaga: FxsQED
MCGPJ: GVMD

dashed line = NLO

McMule plot update :: CMD $\sqrt{s} = 0.7\,\mathrm{GeV}$

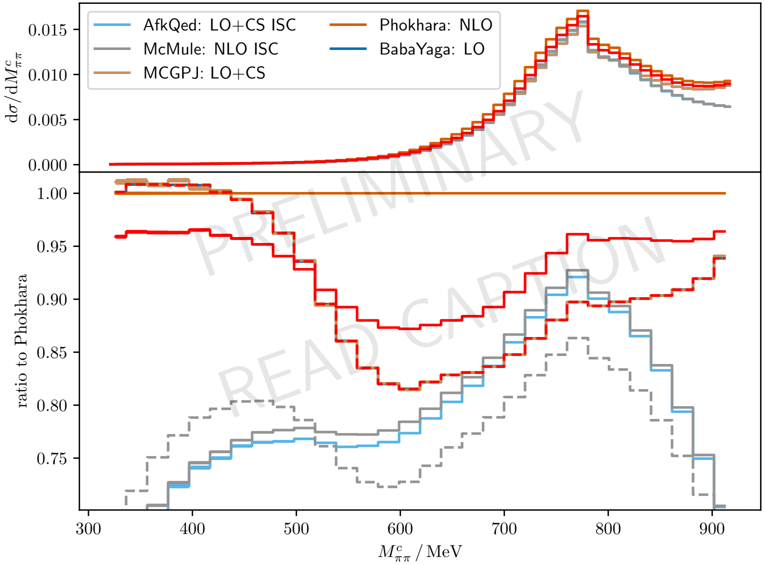


$\theta_{\mathrm{avg}} = (\theta^- - \theta^+ + \pi)/2$

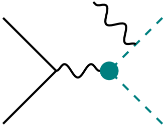


BabaYaga: FxsQED
MCGPJ: GVMD

McMule plot update :: KLOE-LA $\sqrt{s} = 1.02\,\mathrm{GeV}$



cut on M_{trk}

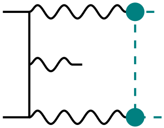


Phokhara: FxsQED
MCGPJ: FxsQED

dashed line = LO

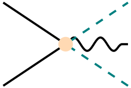
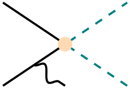
towards $ee \to \pi\pi\gamma$ and beyond

next McMule pion goal: [David's talk, Sara's talk]
$\sim q_e^3 q_\pi^2$ for $ee \to \pi\pi\gamma$ at NLO with FsQED (+ NNLO ISC)



Disperon QED
$=$ OpenLoops $\oplus$ Disperon EFT $\oplus$ universal threshold subtraction

1. We can rely on OpenLoops :: stability & speed
2. The EFT is also valid for $ee \to \pi\pi\gamma$. $\partial_\mu \to D_\mu$ $\Longrightarrow$

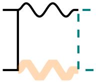
 $\Longrightarrow$ just calculate 

3. Threshold singularities are under control. now

3. Threshold singularities are under control.

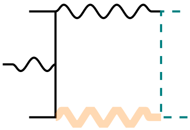
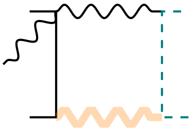
What we know

- $ee \to \pi\pi$ single-dispersive box has a singularity at $S = s \implies$ counterterm

$$\int \mathrm{d}S \left(\right.$$

$$\left. - \frac{1}{S}\left(\frac{s}{S-s}\right)^{1+2\epsilon} f(s,t)\right) + \left(\int \mathrm{d}S \frac{1}{S}\left(\frac{s}{S-s}\right)^{1+2\epsilon}\right) f(s,t)$$

What about $ee \to \pi\pi\gamma$?

- Landau equations $\implies S = s - s_{15} - s_{25}$ for single-dispersive box
- 'Cayley reduction'


$$= \frac{1}{\text{non-vanishing}}$$

$+$ all others

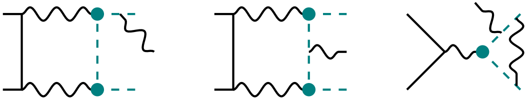
$\implies$ singularities are the same for $ee \to \pi\pi\gamma \implies$ 'universal' subtraction

Disperon QED

$=$ OpenLoops $\oplus$ Disperon EFT $\oplus$ universal threshold subtraction

- will provide $q_e^3 q_\pi^2$ for FsQED $ee \to \pi\pi\gamma$ @ NLO
- will provide VP contributions for $ee \to \mu\mu\gamma$ @ NNLO [David's talk]
  $\Longrightarrow$ speed up process & stable evaluation

- FsQED $ee \to \pi\pi\gamma$ @ NLO beyond $q_e^3 q_\pi^2$ – What makes sense ??



- extend to proton-poles :: two-photon exchange for $ep \to ep$??

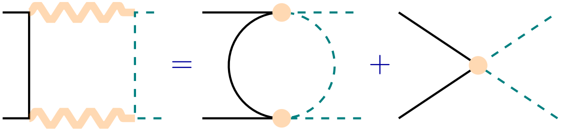
f.l.t.r.: S.Kollatzsch (Zurich & PSI), A.Signer (Zurich & PSI), V.Sharkovska (Zurich & Mainz), S.Gündogdu (Zurich & PSI), D.Moreno (PSI), A.Coutinho (IFIC), Y.Ulrich (Liverpool), D.Radic (Zurich & PSI), L.Naterop (Zurich & PSI), M.Rocco (Turin)
not shown: F.Hagelstein (Mainz), N.Schalch (Oxford), P.Banerjee (Cosenza), M.Ronchi (Mainz), Y.Fang (ETH), P.Wahlen (ETH), R.Krolzig (Zurich)

McMule

mule-tools.gitlab.io

Pion Disperon EFT II

disperons $S \geq \Lambda \gg s$ described by an 'EFT' up to dimension 8 [MSc thesis Y. Fang]



$$+\,\mathcal{O}\left(\frac{1}{S^3}\right)$$

$$\mathcal{L}_{d_1 d_2} = \mathcal{L}_{\gamma d_{1,2}} + \sim \int_\Lambda^\infty \mathrm{d}S_1 \int_\Lambda^\infty \mathrm{d}S_2 \frac{\mathrm{Im}F_\pi(S_1)\mathrm{Im}F_\pi(S_2)}{\pi^2 S_1 S_2} \left\{ \frac{\cdots}{S_1 - S_2} \left[(\bar{\psi} i \not{D} \psi)\pi^\dagger \pi\right] + \ldots \right\} + \mathcal{O}\left(\frac{1}{S^3}\right)$$