PSI Center for Neutron and Muon Sciences

Universität Zürich[UZH]

# Towards $e^+e^- \to \mu^+\mu^-\gamma$ at NNLO

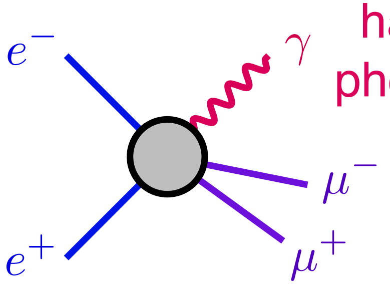
## Workshop on Radiative Corrections and Monte Carlo simulations for electron-positron collisions

David Radic, PSI / UZH

Pisa, 08 May 2025

McMule

# Accuracy requirements for $e^+e^- \to \mu^+\mu^-\gamma$



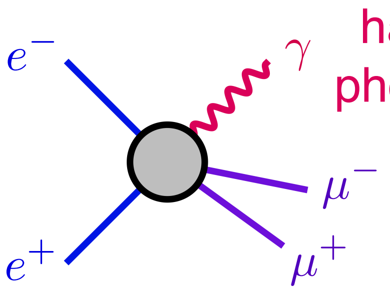

hard photon

radiative corrections are vital!

$$[ee \to \mu\mu\gamma \text{ @NLO}] \subset [ee \to \mu\mu \text{ @NNLO}]$$

ISC, FSC & mixed with full mass dependence

# Accuracy requirements for $e^+e^- \to \mu^+\mu^-\gamma$




hard photon

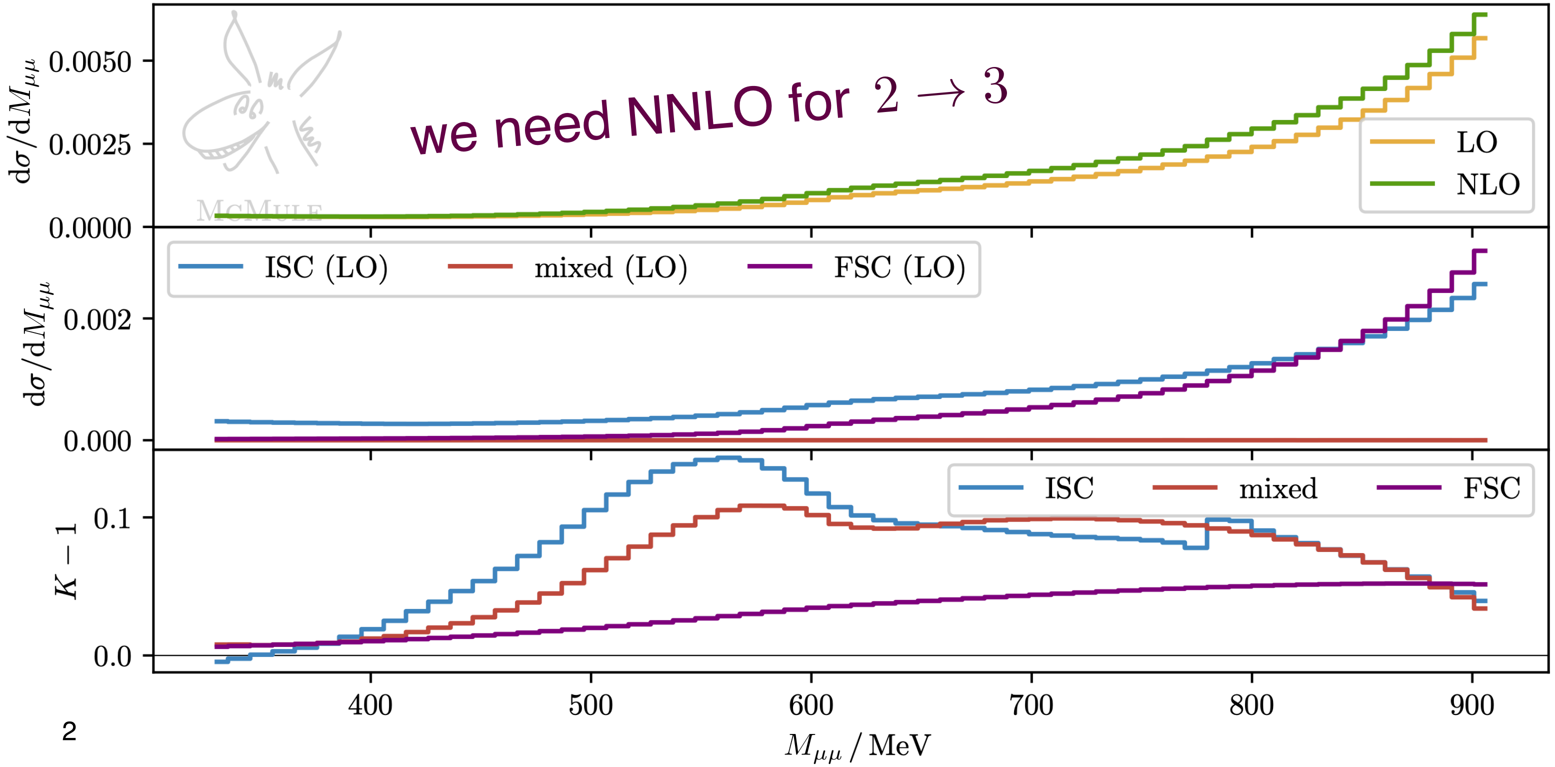
radiative corrections are vital!

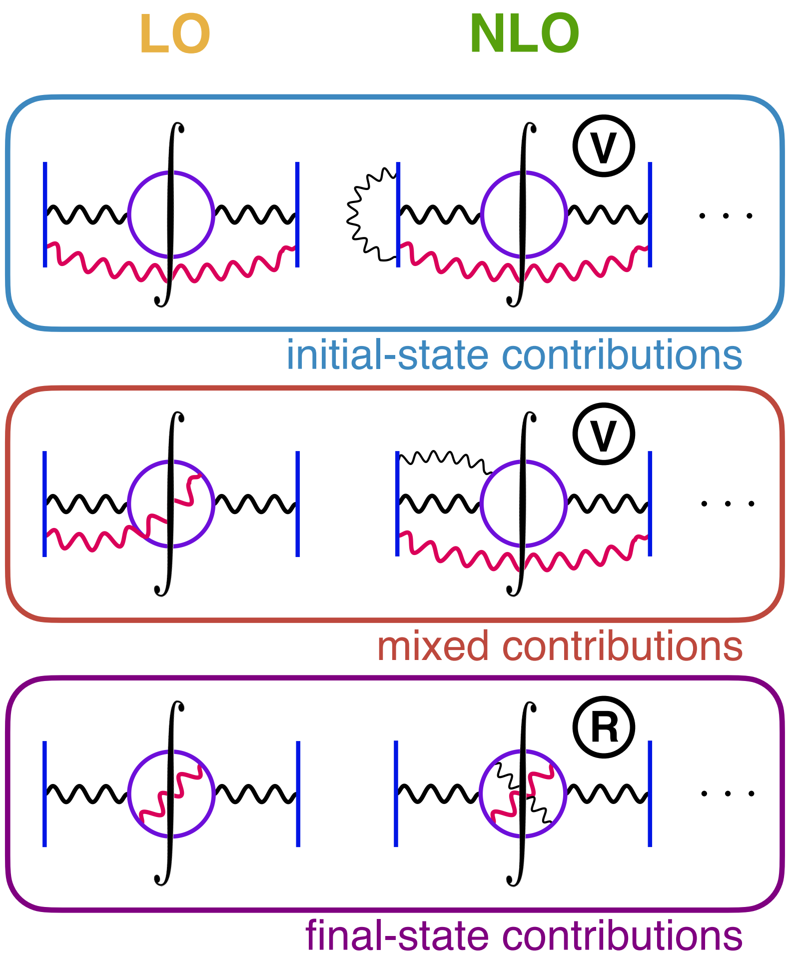
$$[ee \to \mu\mu\gamma \text{ @NLO}] \subset [ee \to \mu\mu \text{ @NNLO}]$$

ISC, FSC & mixed with full mass dependence

$\sim$ KLOE : $\sqrt{s} = 1.02\,\text{GeV}$ ;
$50^\circ \le \theta^\pm, \theta_\gamma \le 130^\circ$ ;
$E_\gamma > 20\,\text{MeV}$

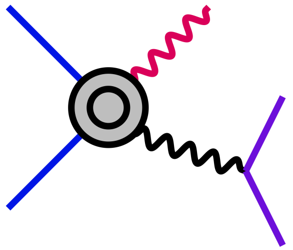
$e^-e^+ \to \mu^-\mu^+\gamma$ @ LA (tagged)

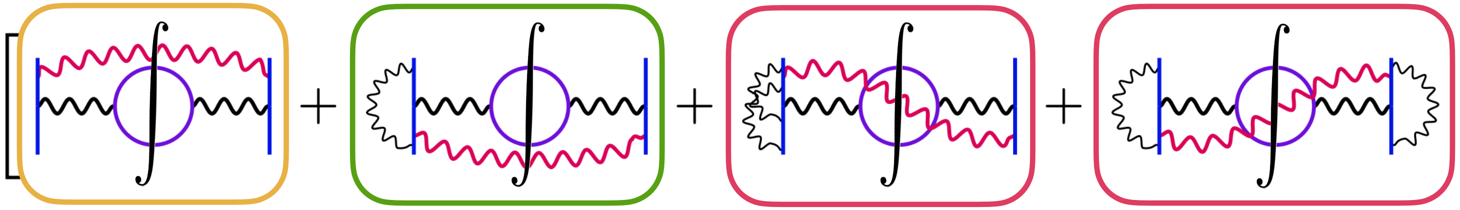


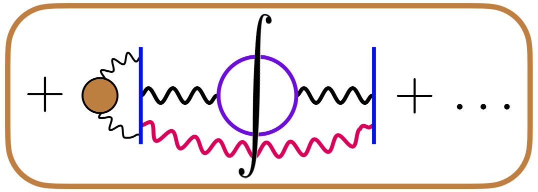



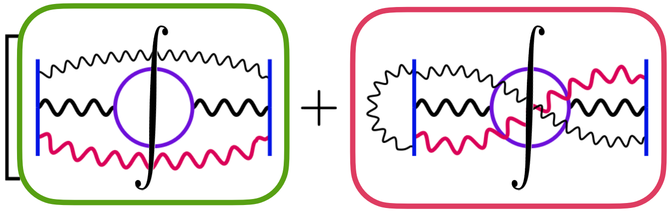
# Anatomy of the NNLO calculation

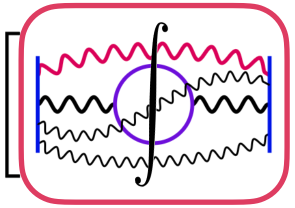
**first:** focus on initial-state corrections for $e^+e^- \to \gamma\,[\gamma^* \to \mu^+\mu^-]$ (ISR)

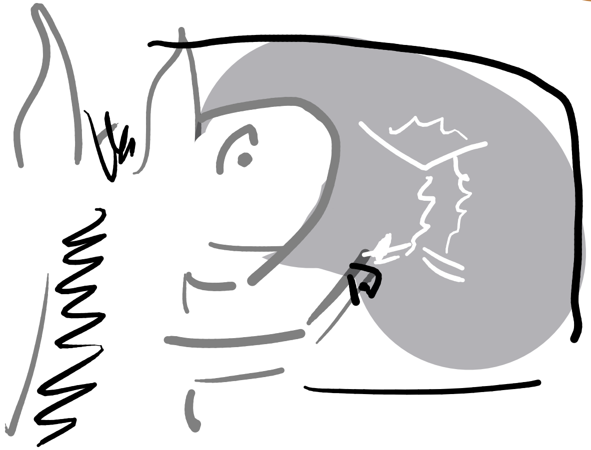


$\sigma_2 = \int \mathrm{d}\Phi_3 \Big[$  $+ \ \dots \Big]$



$+ \int \mathrm{d}\Phi_4 \Big[$  $+ \ \dots \Big]$

$+ \int \mathrm{d}\Phi_5 \Big[$  $+ \ \dots \Big]$



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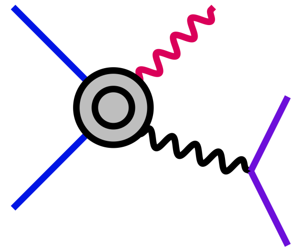
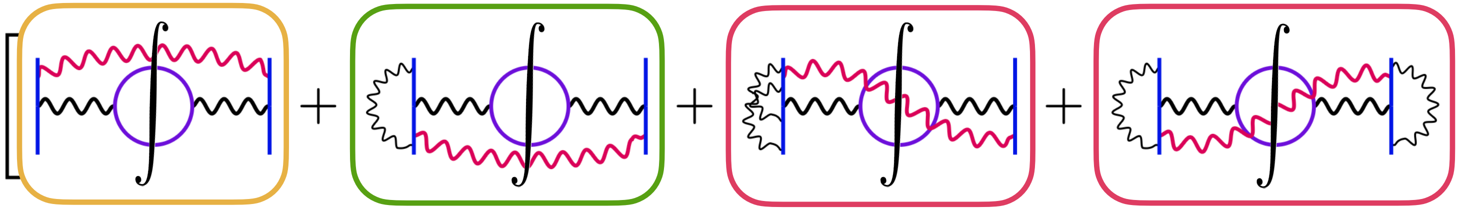
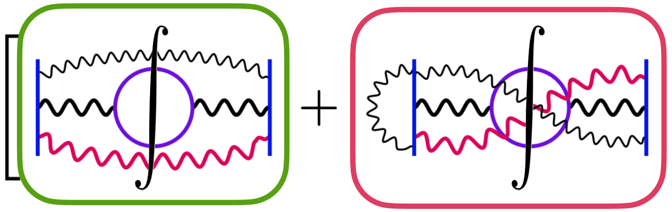
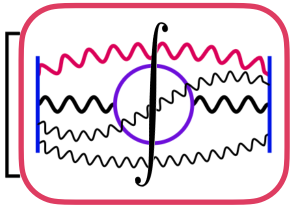
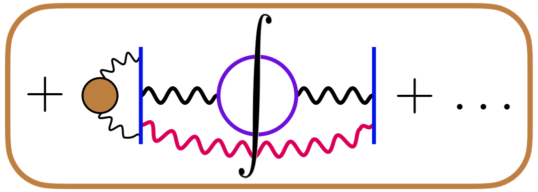


$\sigma_2 = \int \mathrm{d}\Phi_3$ [  + ... ]

$+ \int \mathrm{d}\Phi_4$ [  + ... ]

$+ \int \mathrm{d}\Phi_5$ [  + ... ]



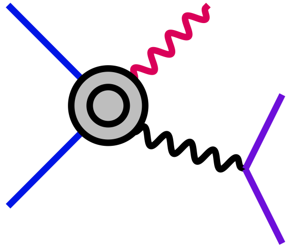
**challenges:**

- divergent phase space integration
- numerical instabilities
- virtual amplitudes with finite masses
- VP contributions

# Anatomy of the NNLO calculation

FKS$^\ell$ subtraction scheme

PSI



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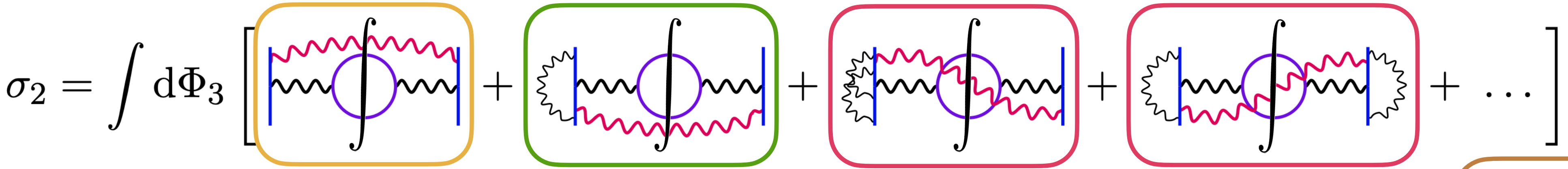

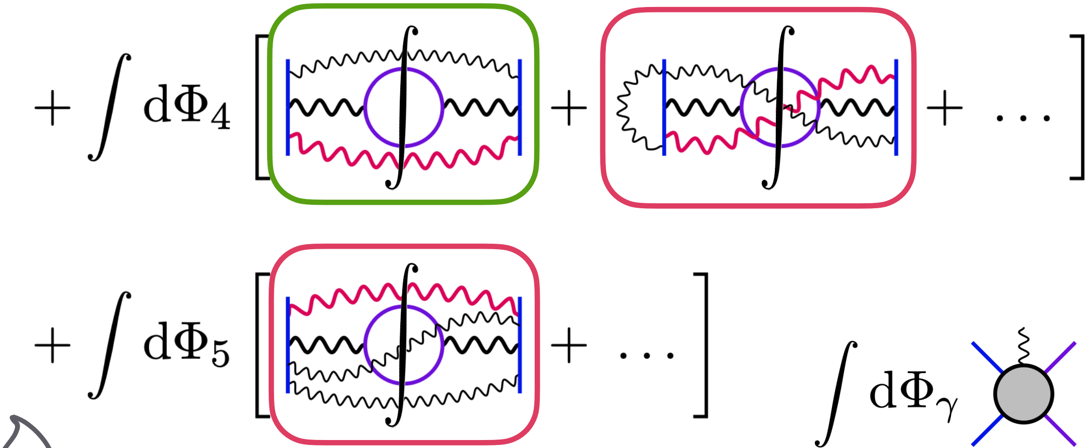

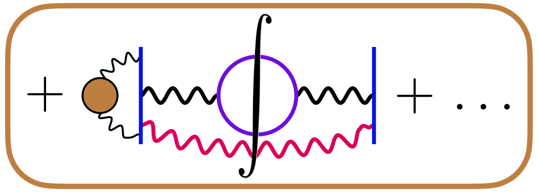

**challenges:**

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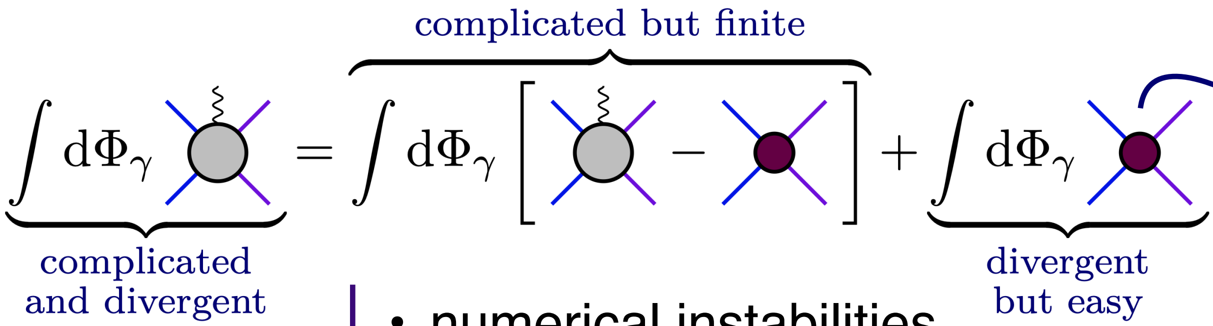
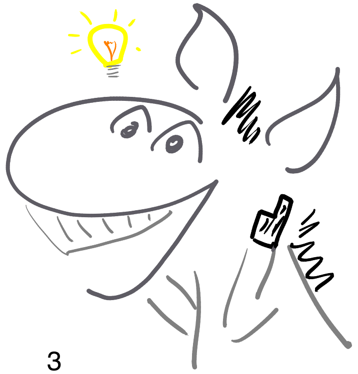



combine with virtual corrections $\Rightarrow$ finite

- numerical instabilities
- virtual amplitudes with finite masses
- VP contributions

subtract universal CT [Engel, Signer, Ulrich 19]

unphysical cut parameter $0 < \xi_c \lesssim 1$

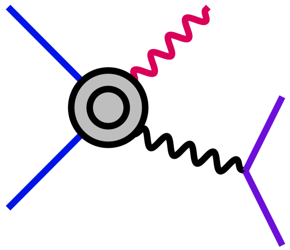


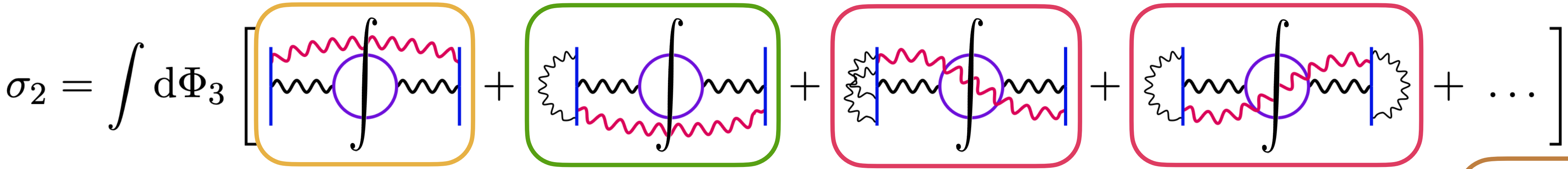
# Anatomy of the NNLO calculation

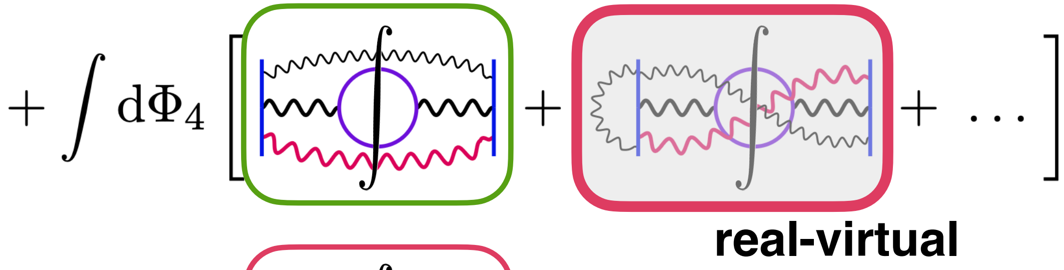
next-to-soft expansion

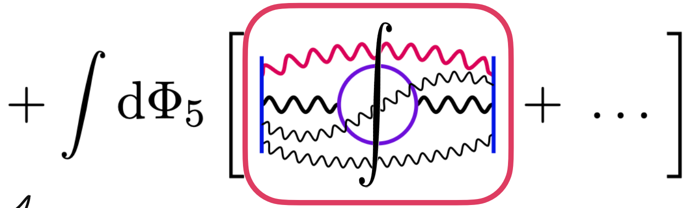
[Sara's talk]

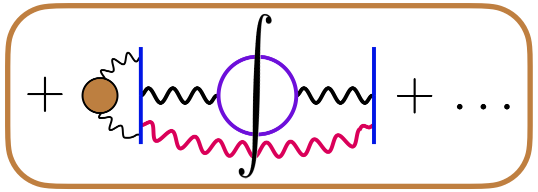
**first:** focus on initial-state corrections for $e^+e^- \to \gamma\,[\gamma^* \to \mu^+\mu^-]$ (ISR)





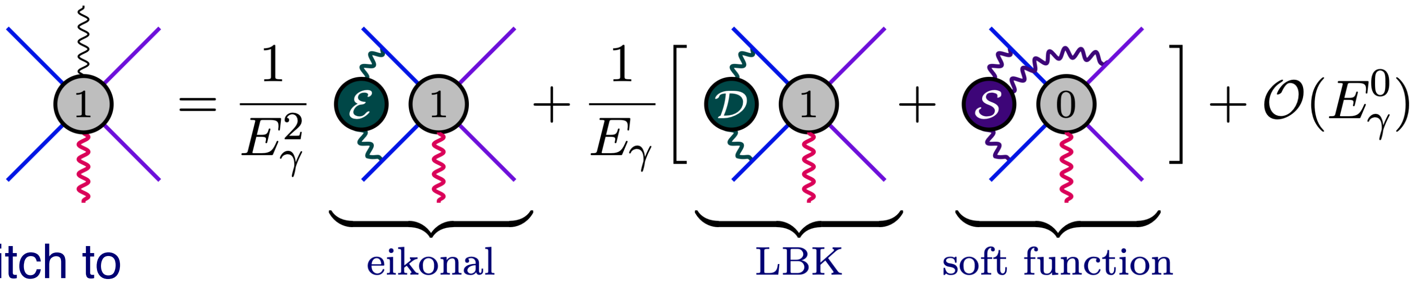




**challenges:**

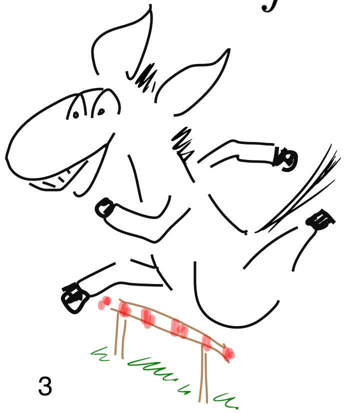
- divergent phase space integration
- numerical instabilities




expand for small $E_\gamma$ and switch to approximation [Low 58; Burnett, Kroll 67] [Engel, Signer, Ulrich 21; Engel 23]

$\Rightarrow$ significant speed-up

- virtual amplitudes with finite masses
- VP contributions

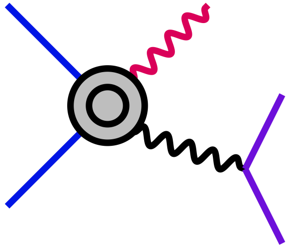


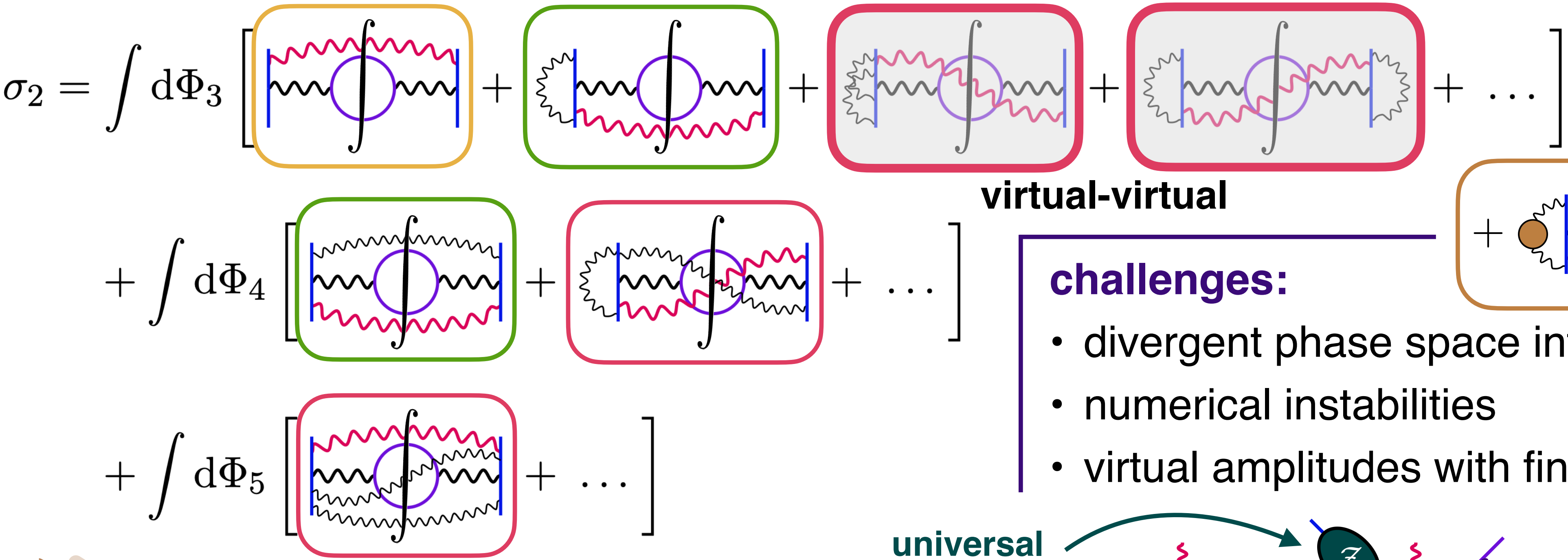
# Anatomy of the NNLO calculation

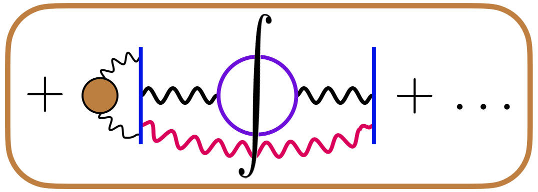
massification

[Sara's talk]

**first:** focus on initial-state corrections for $e^+e^- \to \gamma\,[\gamma^* \to \mu^+\mu^-]$ (ISR)



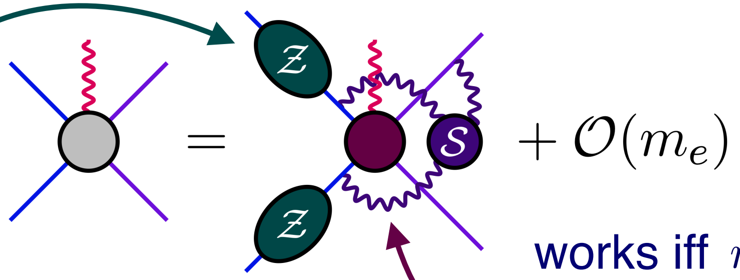



**challenges:**

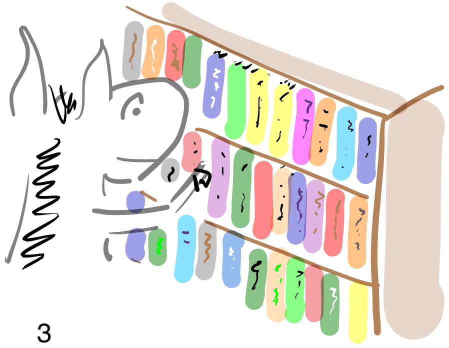
- divergent phase space integration
- numerical instabilities
- virtual amplitudes with finite masses

**universal** massification constant




works iff $m_e^2 \ll s_{ij}$

**massless**



massless amplitude ✓

$\Rightarrow$ convert $1/\varepsilon \to \log(m_e^2/Q^2)$

based on SCET & method of regions

[Penin 06; Mitov, Moch 06; Becher, Melnikov 07]
[Engel, Gnendiger, Signer, Ulrich 21]

- VP contributions

# Anatomy of the NNLO calculation

disperon QED

PSI

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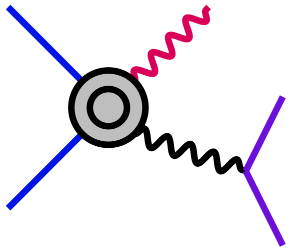
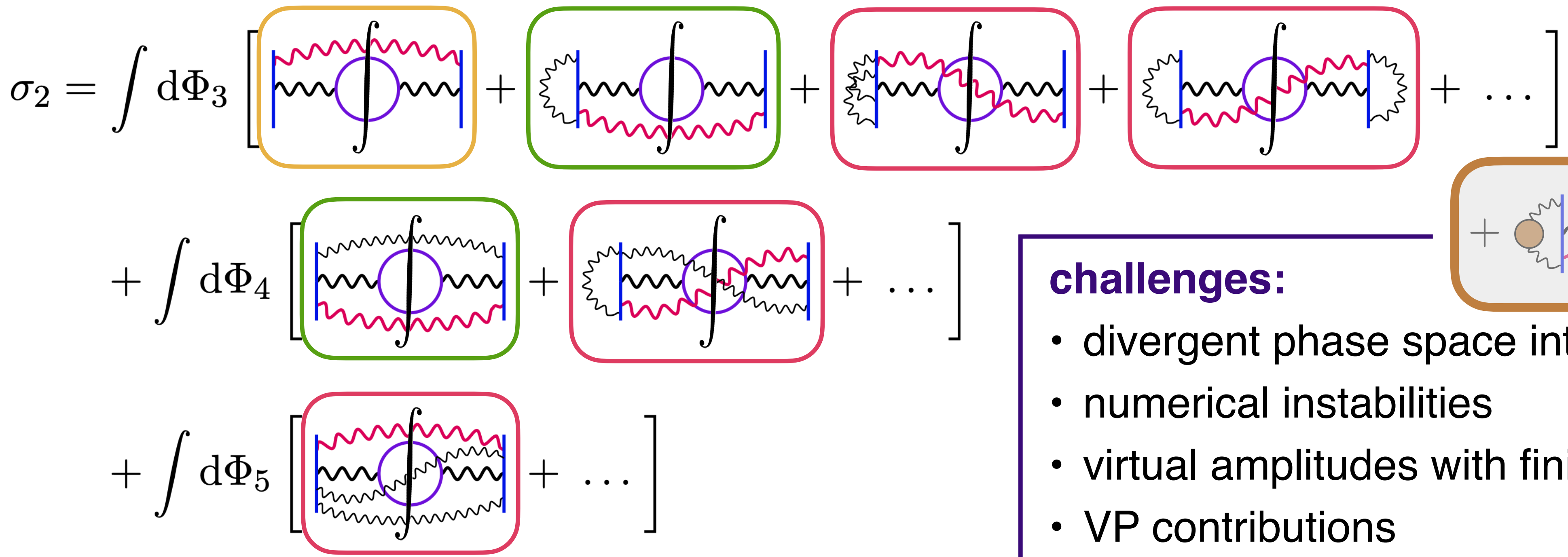
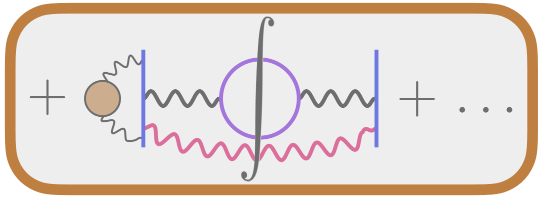





**VP**




**challenges:**

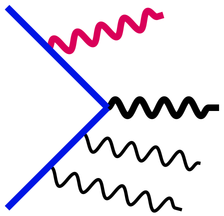
- divergent phase space integration
- numerical instabilities
- virtual amplitudes with finite masses
- VP contributions

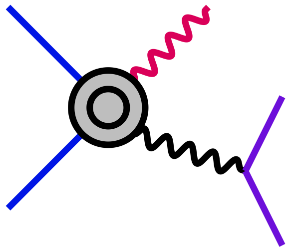
[Sophie's talk]

not yet included

# The main bottlenecks

**real-real** corrections for **large $\sqrt{s}$** ~ BES- & B-like scenario





→ numerical instabilities

$\Rightarrow$ collinear subtraction? [Dittmaier et al. 08]

on our to-do list **?**

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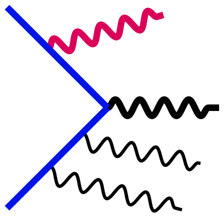
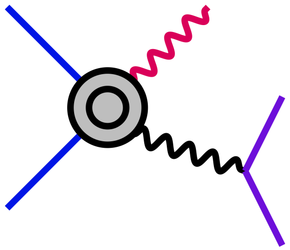
**real-real** corrections for **large** $\sqrt{s}$ ~ BES- & B-like scenario



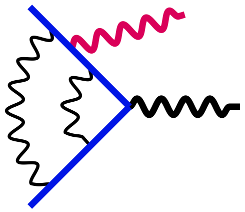


↳ numerical instabilities

$\Rightarrow$ collinear subtraction? [Dittmaier et al. 08]

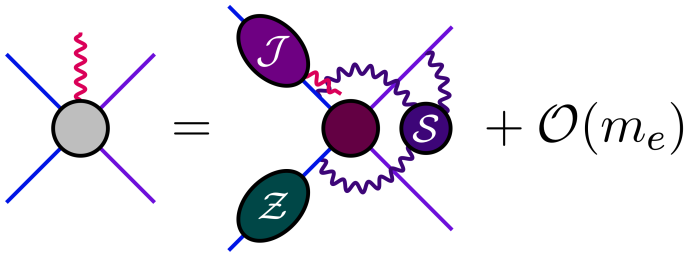
on our to-do list **?**

**hard-collinear emission** $\theta_\gamma \to 0$, i.e. $m_e^2 \sim s_{e\gamma}$ ~ KLOE-like SA scenario

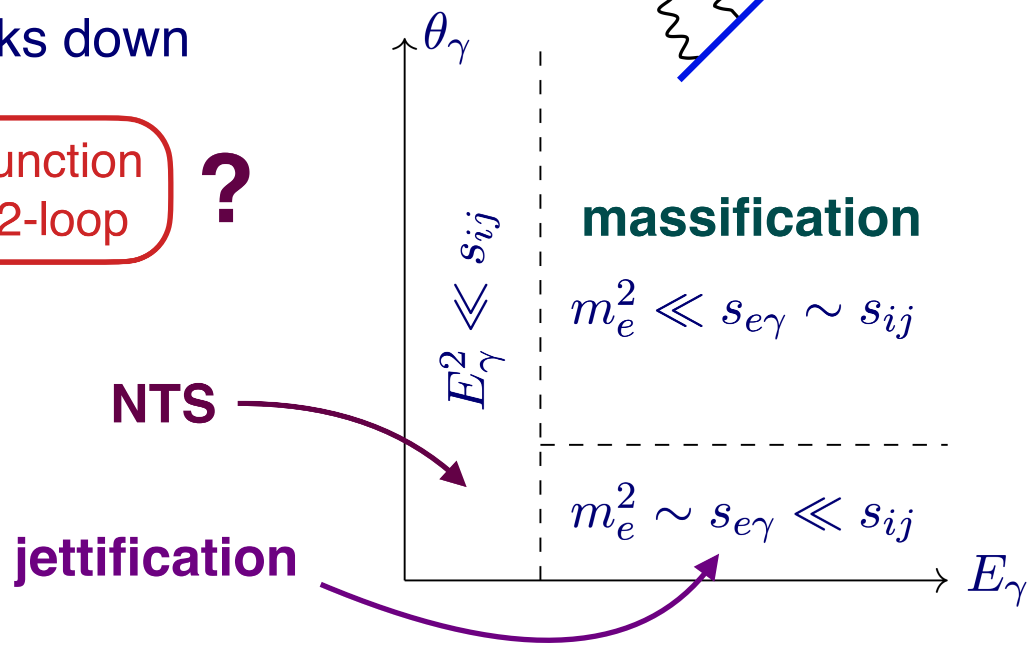


↳ massification for **virtual-virtual** corrections breaks down

$\Rightarrow$ **jettification** [Sara's talk]

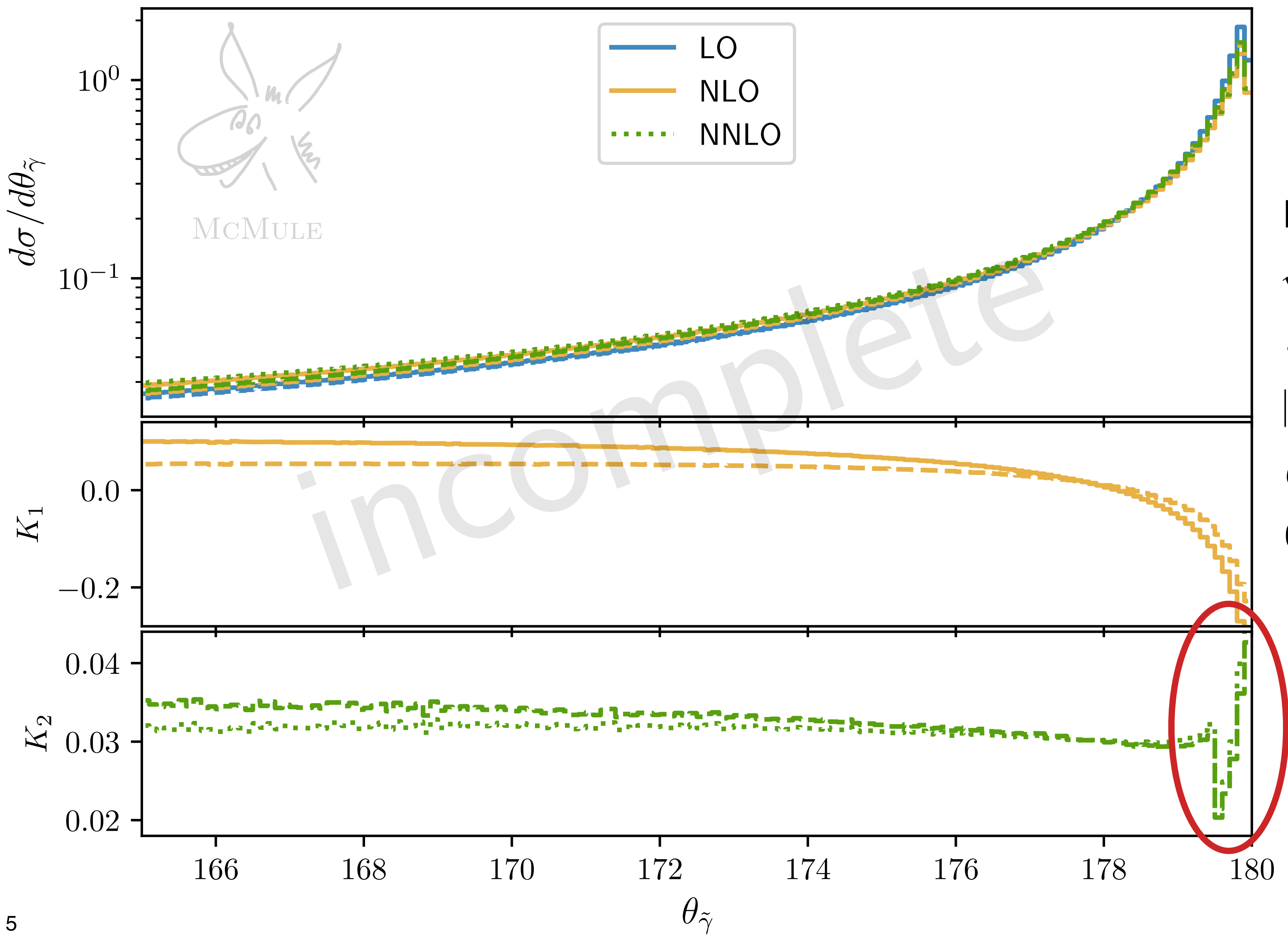


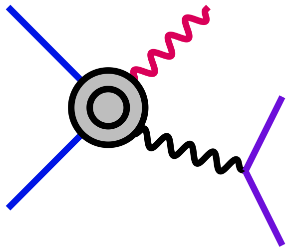

massive jet function unknown at 2-loop **?**




… but no problems for ~ KLOE-like LA scenario

# Results



**KLOE-SA**

$\sqrt{s} = 1.02\,\mathrm{GeV}\,;$

$50^\circ \leq \theta^\pm \leq 130^\circ\,;$

$|p_z^\pm| > 90\,\mathrm{MeV} \,\vee\, |p_\perp^\pm| > 160\,\mathrm{MeV}\,;$

$\theta_{\tilde\gamma} \leq 15^\circ \,\vee\, \theta_{\tilde\gamma} \geq 165^\circ\,;$

$0.35\,\mathrm{GeV}^2 \leq M_{\mu\mu}^2 \leq 0.95\,\mathrm{GeV}^2$

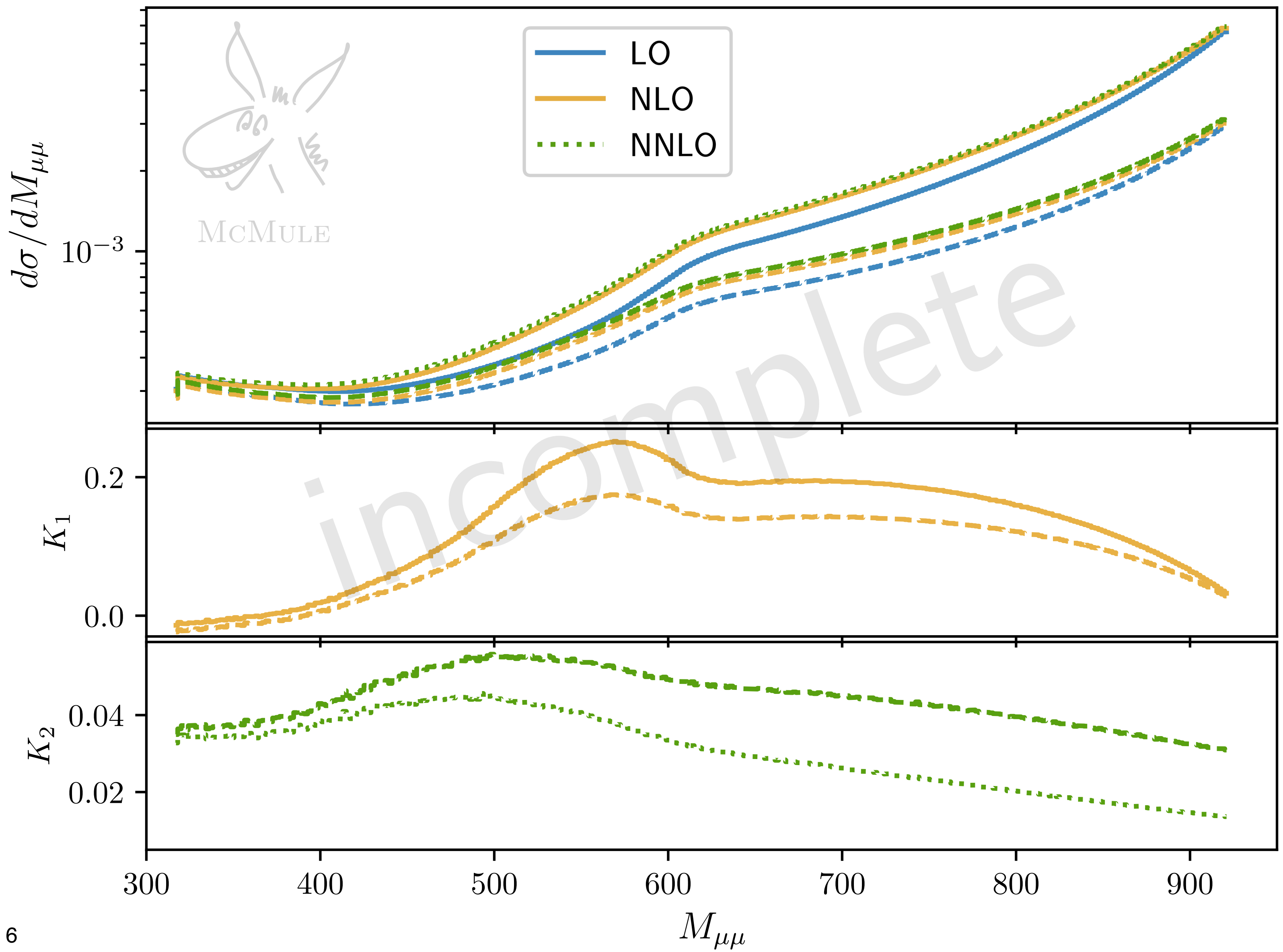
hard-collinear emission

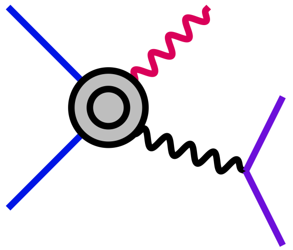
cut $\theta_{\tilde\gamma} \leq 179.5^\circ$ for virtual-virtual corrections

$\Rightarrow$ error, $\xi_c$ dependence (VV missing)

[Sara's talk]

# Results



**KLOE-LA**

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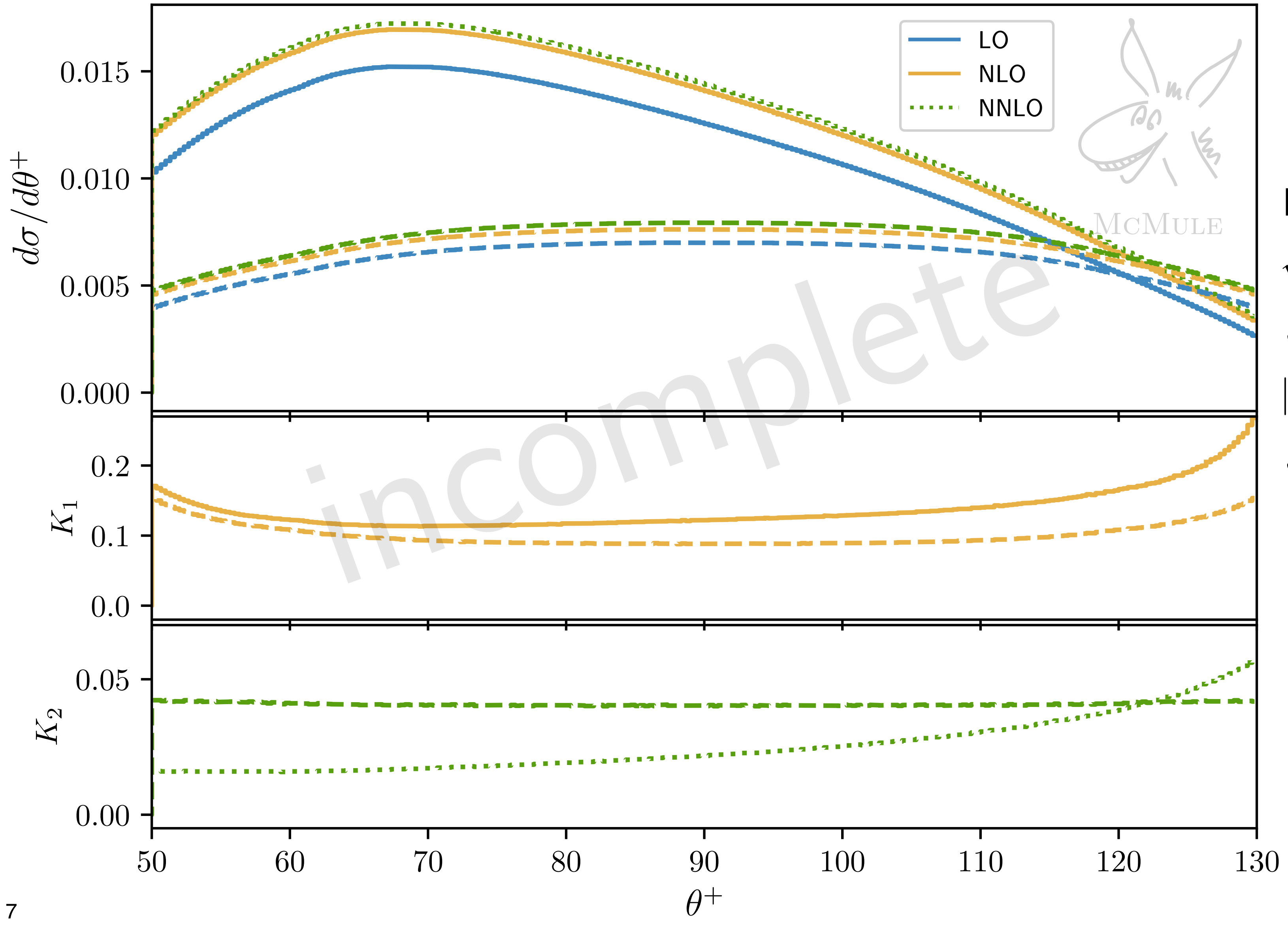
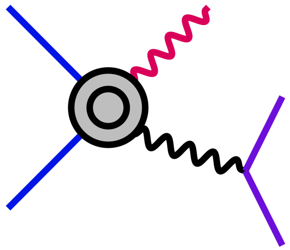
$50^\circ \leq \theta_\gamma \leq 130^\circ \;\wedge\; E_\gamma > 20\,\mathrm{MeV}\,;$

$0.1\,\mathrm{GeV}^2 \leq M_{\mu\mu}^2 \leq 0.85\,\mathrm{GeV}^2$

# Results



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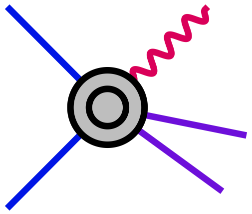
$0.1\,\mathrm{GeV}^2 \le M_{\mu\mu}^2 \le 0.85\,\mathrm{GeV}^2$

# Next steps

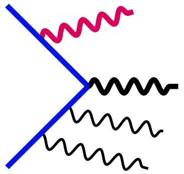
full $e^+e^- \to \mu^+\mu^-\gamma$ (ISC + FSC + mixed) with $m_e^2 \sim m_\mu^2 \ll s_{ij}$ up to NNLO



→ bosonic corrections: no theoretical showstoppers

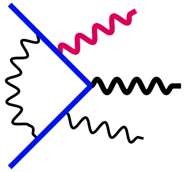
NLO solved (matrix elements with full mass dependence from OL)

**current state @ NNLO:** ISC ✓ ⇒ FSC ✓

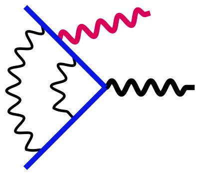


OL

not ideal

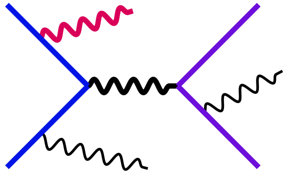


OL + NTS



massification ( $m_e$ / $m_\mu$ )
[Badger et al. 23]

**mixed:**



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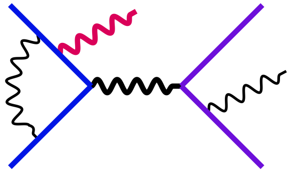

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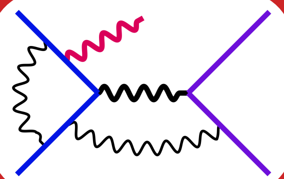
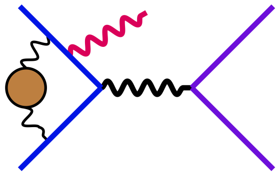


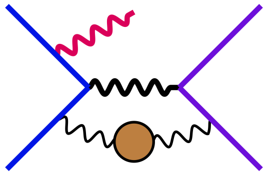
**not yet included**

massification ( $m_e$ & $m_\mu$ )

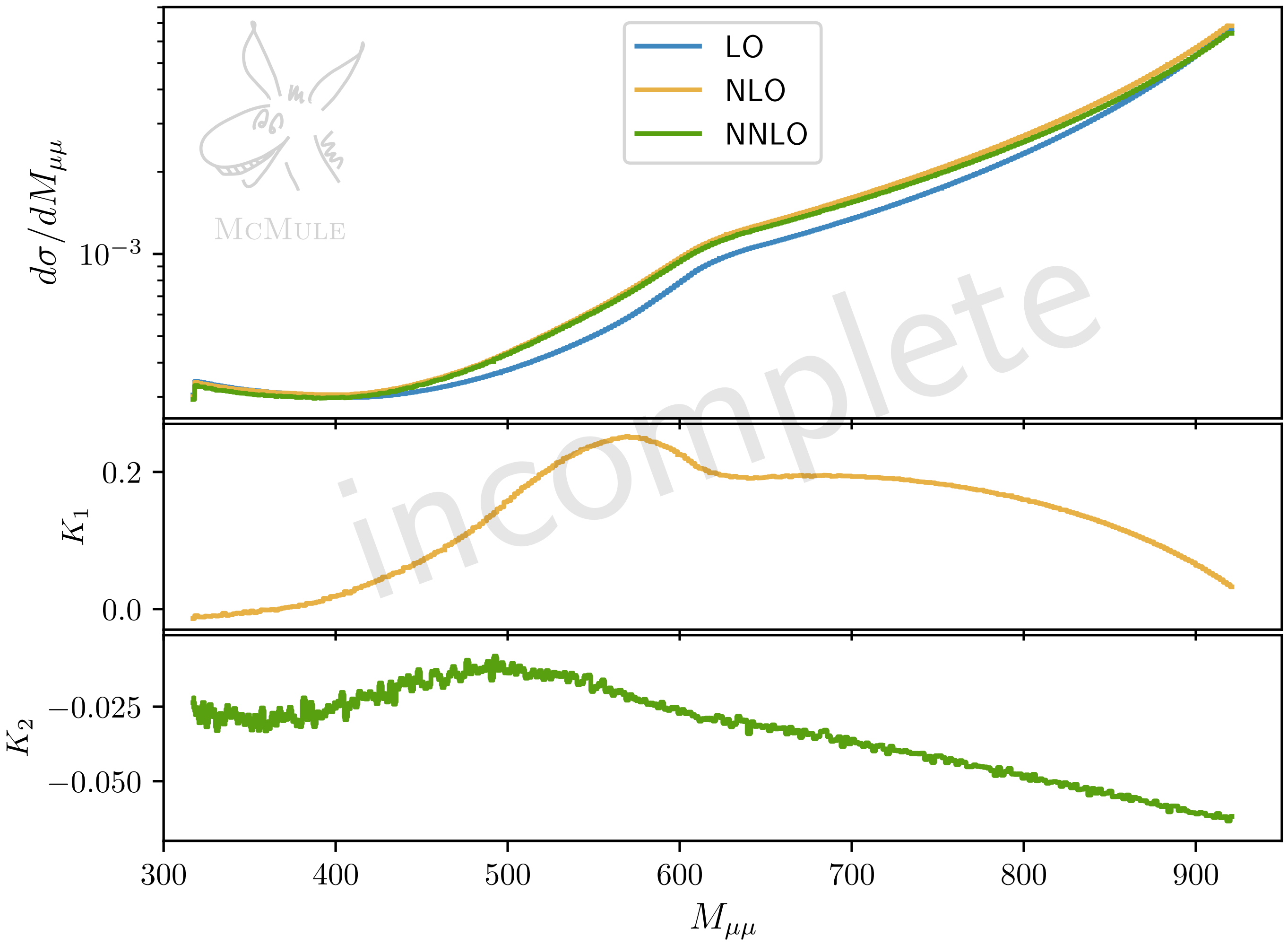
amplitudes from $pp \to 2j + \gamma$ [Badger et al. 23]

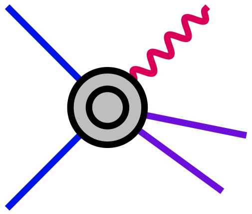
**still missing:** fermionic contributions



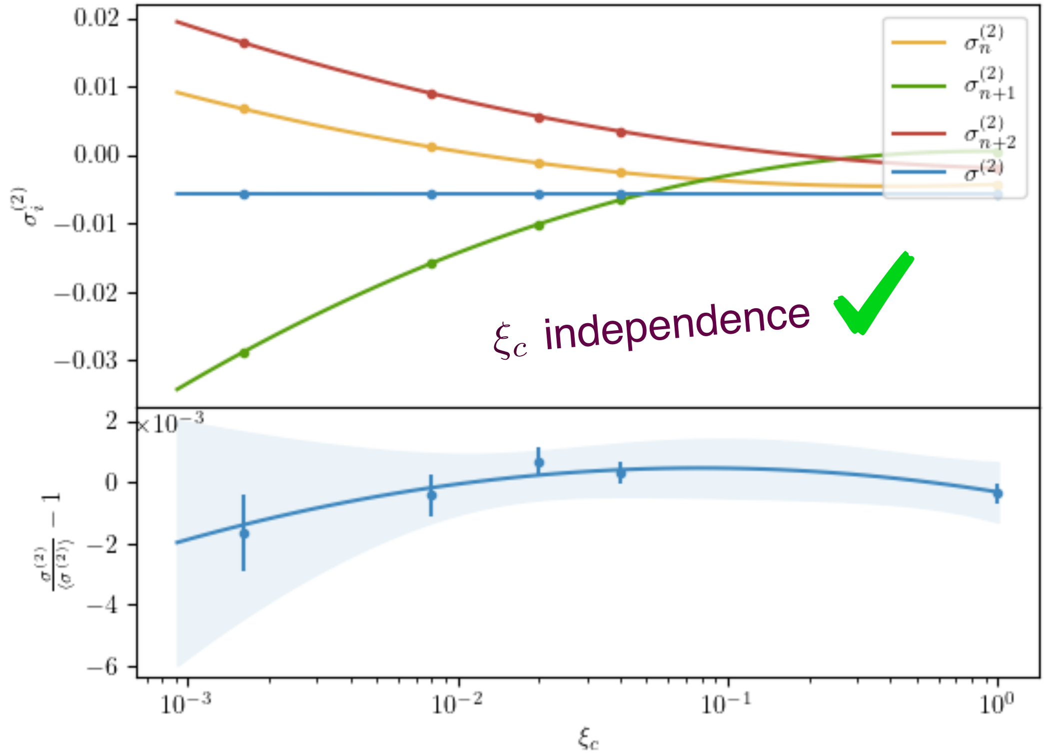


# Results for KLOE-LA: full NNLO w/o virtual-virtual



check stability of full RR & RV

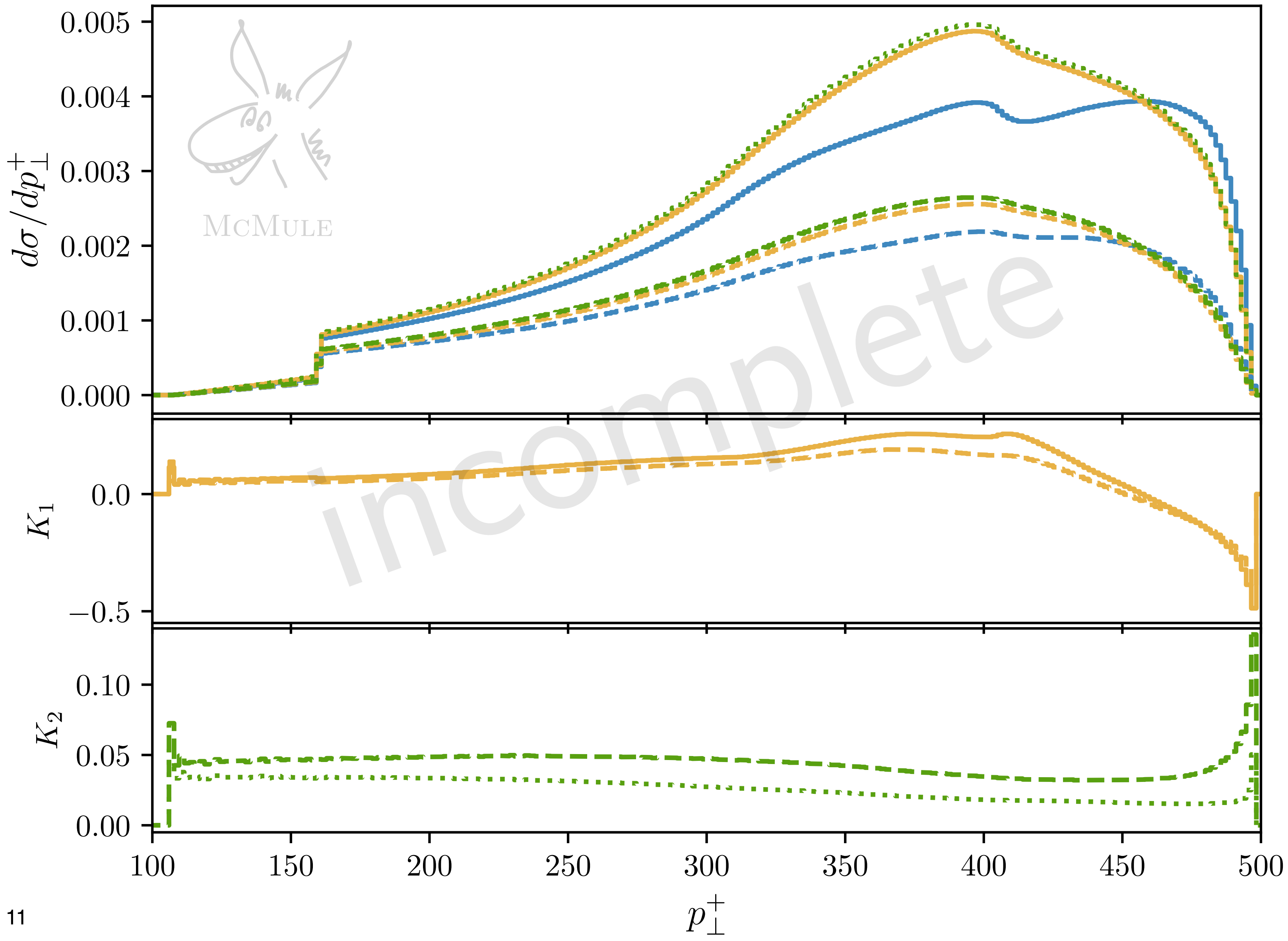


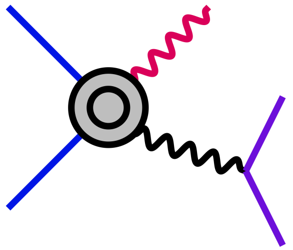
# Conclusions

- ISC / FSC at NNLO under control for $m^2_{e/\mu} \ll s_{ij}$ ✓
  - results surprisingly large (?) (analysis not yet completed)
- full NNLO: first checks for RR & RV successful
- full $ee \to \mu\mu\gamma$ at NNLO fairly doable in the limit $m_e^2 \sim m_\mu^2 \ll s_{ij}$
  - However, …
    - numerical instabilities for BES- & B-like energies
    - massification of virtual-virtual corrections does not work for hard-collinear emission

  $\Rightarrow$ needs to be investigated further

# Appendix



**KLOE-LA**

$\sqrt{s} = 1.02\,\text{GeV}\,;$

$50^\circ \le \theta^\pm \le 130^\circ\,;$

$|p_z^\pm| > 90\,\text{MeV} \,\vee\, |p_\perp^\pm| > 160\,\text{MeV}\,;$

$50^\circ \le \theta_\gamma \le 130^\circ \,\wedge\, E_\gamma > 20\,\text{MeV}\,;$

$0.1\,\text{GeV}^2 \le M_{\mu\mu}^2 \le 0.85\,\text{GeV}^2$