

PSI Center for Neutron and
Muon Sciences



**Universität
Zürich** ^{UZH}

Towards $e^+e^- \rightarrow \mu^+\mu^-\gamma$ at NNLO

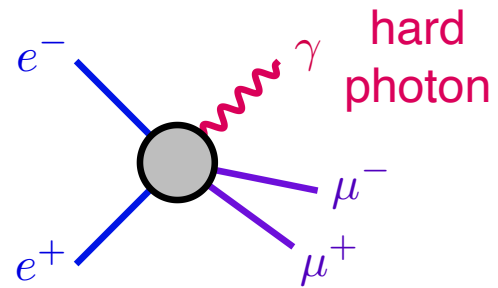
**Workshop on Radiative Corrections and Monte Carlo
simulations for electron-positron collisions**

David Radic, PSI / UZH
Pisa, 08 May 2025



McMule

Accuracy requirements for $e^+e^- \rightarrow \mu^+\mu^-\gamma$

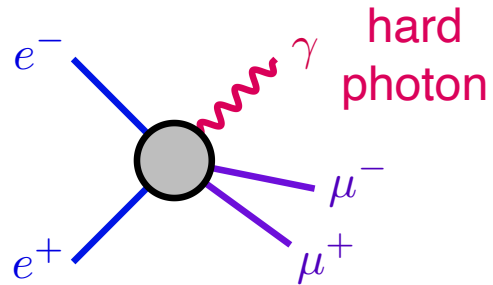


radiative corrections are vital!

$$[ee \rightarrow \mu\mu\gamma \text{ @NLO}] \subset [ee \rightarrow \mu\mu \text{ @NNLO}]$$

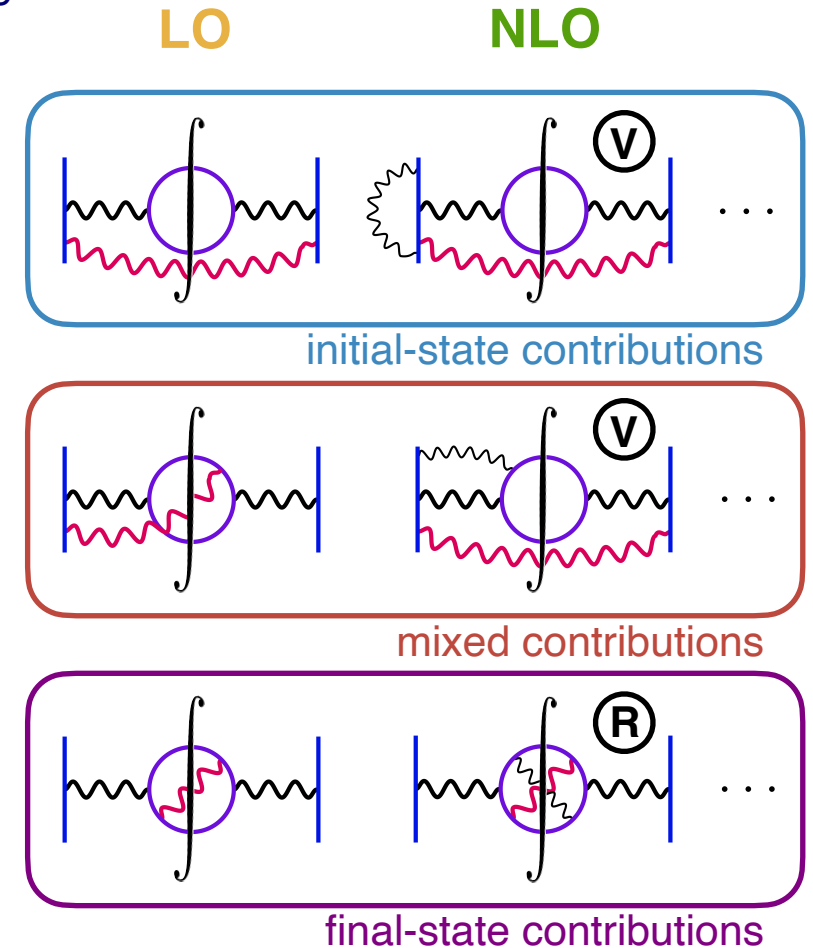
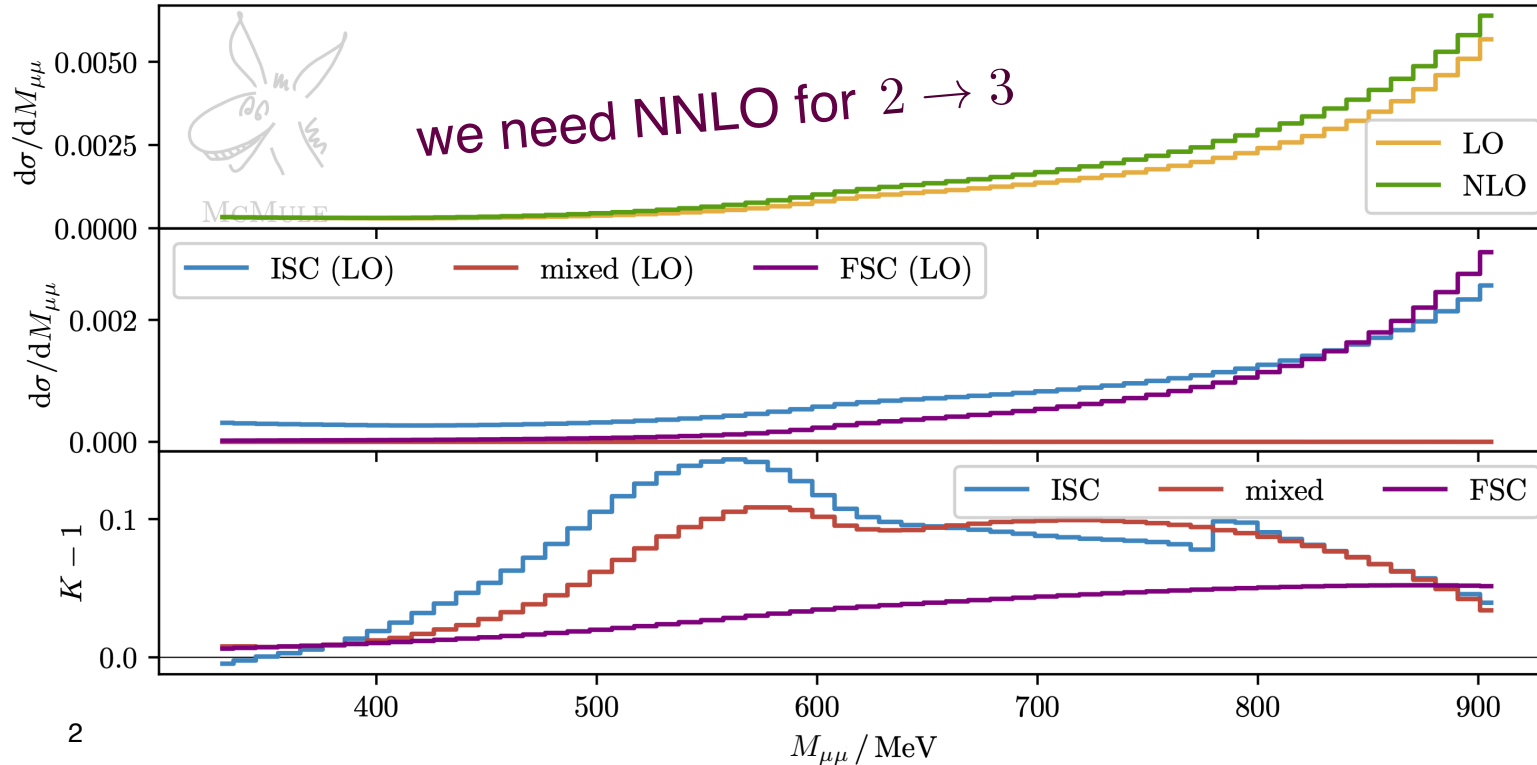
ISC, FSC & mixed with full mass dependence

Accuracy requirements for $e^+e^- \rightarrow \mu^+\mu^-\gamma$



radiative corrections are vital!
 $[ee \rightarrow \mu\mu\gamma \text{ @NLO}] \subset [ee \rightarrow \mu\mu \text{ @NNLO}]$
 ISC, FSC & mixed with full mass dependence

\sim KLOE : $\sqrt{s} = 1.02 \text{ GeV}$;
 $50^\circ \leq \theta^\pm, \theta_\gamma \leq 130^\circ$;
 $E_\gamma > 20 \text{ MeV}$
 $e^-e^+ \rightarrow \mu^-\mu^+\gamma \text{ @ LA (tagged)}$



Anatomy of the NNLO calculation



first: focus on initial-state corrections for $e^+e^- \rightarrow \gamma [\gamma^* \rightarrow \mu^+\mu^-]$ (ISR)

$$\begin{aligned}
 \sigma_2 = & \int d\Phi_3 \left[\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} + \dots \right] \\
 & + \int d\Phi_4 \left[\text{diagram 5} + \text{diagram 6} + \dots \right] \\
 & + \int d\Phi_5 \left[\text{diagram 7} + \dots \right]
 \end{aligned}$$

The diagrams in the equation represent various initial-state radiation (ISR) corrections. Diagram 1 (orange box) is the Born process. Diagrams 2-4 (green, red, red boxes) show single-photon emission from the incoming electron or positron lines. Diagrams 5-6 (green, red boxes) show two-photon emission. Diagram 7 (red box) shows a more complex higher-order correction. To the right, a separate diagram shows a central interaction vertex (grey circle) with four external lines: two blue lines and two purple lines, and a red wavy line. Below this, another diagram (orange box) shows a similar vertex with a brown dot on one of the blue lines. A hand-drawn sketch of a person's head with a wavy line next to it is also present.

Anatomy of the NNLO calculation



first: focus on initial-state corrections for $e^+e^- \rightarrow \gamma [\gamma^* \rightarrow \mu^+\mu^-]$ (ISR)

$$\begin{aligned}
 \sigma_2 = & \int d\Phi_3 \left[\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} + \dots \right] \\
 & + \int d\Phi_4 \left[\text{diagram 5} + \text{diagram 6} + \dots \right] \\
 & + \int d\Phi_5 \left[\text{diagram 7} + \dots \right]
 \end{aligned}$$

The diagrams represent various initial-state radiation (ISR) corrections. Diagram 1 (orange box) shows a photon radiating from the incoming electron line. Diagram 2 (green box) shows a photon radiating from the incoming positron line. Diagram 3 (red box) shows a photon radiating from the internal fermion line. Diagram 4 (red box) shows a photon radiating from the internal fermion line in a different configuration. Diagram 5 (green box) shows a photon radiating from the incoming electron line in a different configuration. Diagram 6 (red box) shows a photon radiating from the incoming positron line in a different configuration. Diagram 7 (red box) shows a photon radiating from the incoming electron line in a different configuration. Diagram 8 (orange box) shows a photon radiating from the incoming electron line in a different configuration. Diagram 9 (grey circle) shows a photon radiating from the internal fermion line in a different configuration. Diagram 10 (orange box) shows a photon radiating from the incoming electron line in a different configuration.

- challenges:**
- divergent phase space integration
 - numerical instabilities
 - virtual amplitudes with finite masses
 - VP contributions



Anatomy of the NNLO calculation

FKS^ℓ subtraction scheme



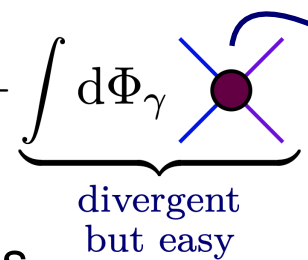
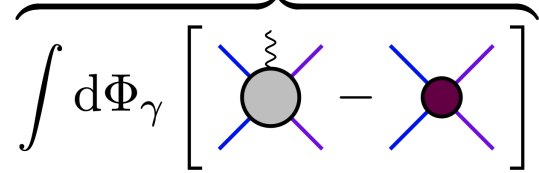
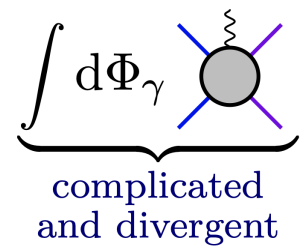
first: focus on initial-state corrections for $e^+e^- \rightarrow \gamma [\gamma^* \rightarrow \mu^+\mu^-]$ (ISR)

$$\sigma_2 = \int d\Phi_3 \left[\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} + \dots \right] + \int d\Phi_4 \left[\text{diagram 5} + \text{diagram 6} + \dots \right] + \int d\Phi_5 \left[\text{diagram 7} + \dots \right]$$

challenges:

- divergent phase space integration

complicated but finite



combine with virtual corrections \Rightarrow finite

- numerical instabilities
- virtual amplitudes with finite masses
- VP contributions

subtract universal CT [Engel, Signer, Ulrich 19]
 unphysical cut parameter $0 < \xi_c \lesssim 1$



Anatomy of the NNLO calculation

next-to-soft expansion



[Sara's talk]

first: focus on initial-state corrections for $e^+e^- \rightarrow \gamma [\gamma^* \rightarrow \mu^+\mu^-]$ (ISR)

$$\sigma_2 = \int d\Phi_3 \left[\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} + \dots \right]$$

$$+ \int d\Phi_4 \left[\text{diagram 5} + \text{diagram 6} + \dots \right]$$

$$+ \int d\Phi_5 \left[\text{diagram 7} + \dots \right]$$

real-virtual

$$= \frac{1}{E_\gamma^2} \left[\text{eikonal} + \frac{1}{E_\gamma} \left[\text{LBK} + \text{soft function} \right] \right] + \mathcal{O}(E_\gamma^0)$$

challenges:

- divergent phase space integration
- numerical instabilities

expand for small E_γ and switch to approximation [Low 58; Burnett, Kroll 67] [Engel, Signer, Ulrich 21; Engel 23]

⇒ significant speed-up



Anatomy of the NNLO calculation

massification



[Sara's talk]

first: focus on initial-state corrections for $e^+e^- \rightarrow \gamma [\gamma^* \rightarrow \mu^+\mu^-]$ (ISR)

$$\sigma_2 = \int d\Phi_3 \left[\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} + \dots \right] + \int d\Phi_4 \left[\text{diagram 5} + \text{diagram 6} + \dots \right] + \int d\Phi_5 \left[\text{diagram 7} + \dots \right]$$

virtual-virtual

challenges:

- divergent phase space integration
- numerical instabilities
- virtual amplitudes with finite masses

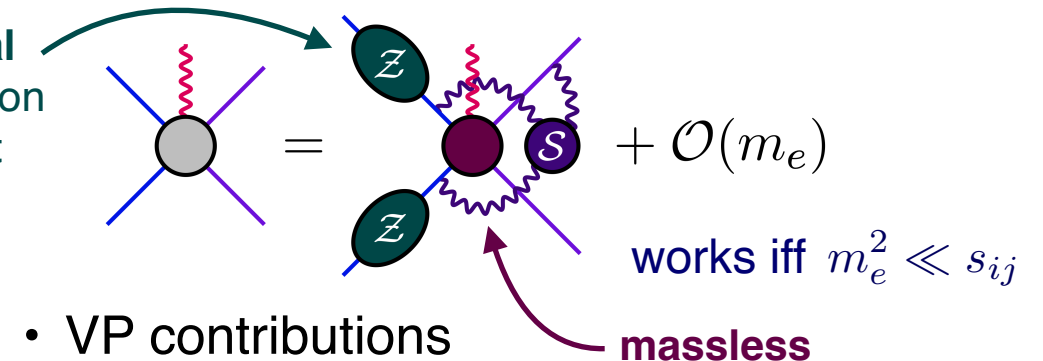


massless amplitude ✓
 \Rightarrow convert $1/\epsilon \rightarrow \log(m_e^2/Q^2)$

based on SCET & method of regions

[Penin 06; Mitov, Moch 06; Becher, Melnikov 07]
 [Engel, Gnendiger, Signer, Ulrich 21]

universal massification constant



- VP contributions

Anatomy of the NNLO calculation

disperon QED



first: focus on initial-state corrections for $e^+e^- \rightarrow \gamma [\gamma^* \rightarrow \mu^+\mu^-]$ (ISR)

$$\sigma_2 = \int d\Phi_3 \left[\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} + \dots \right] + \int d\Phi_4 \left[\text{diagram 5} + \text{diagram 6} + \dots \right] + \int d\Phi_5 \left[\text{diagram 7} + \dots \right]$$

The diagrams are Feynman diagrams for initial-state radiation (ISR) corrections. Diagrams 1-4 are grouped in a box labeled 'VP' (Virtual Photon). Diagram 5 is highlighted in a grey box with an orange border. Diagram 7 is also highlighted in a grey box with an orange border.

- challenges:**
- divergent phase space integration
 - numerical instabilities
 - virtual amplitudes with finite masses
 - VP contributions
- [Sophie's talk]

not yet included



The main bottlenecks

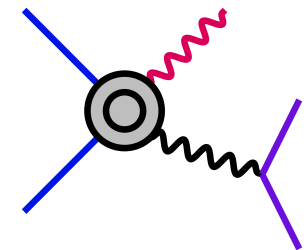
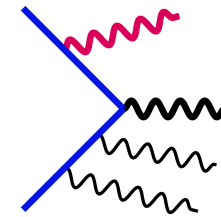
real-real corrections for large \sqrt{s} ~ BES- & B-like scenario

→ numerical instabilities

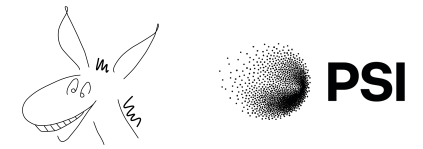
⇒ collinear subtraction? [Dittmaier et al. 08]

on our to-do list

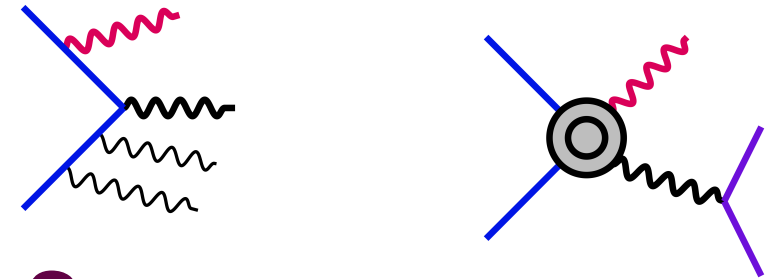
?



The main bottlenecks



real-real corrections for large \sqrt{s} \sim BES- & B-like scenario

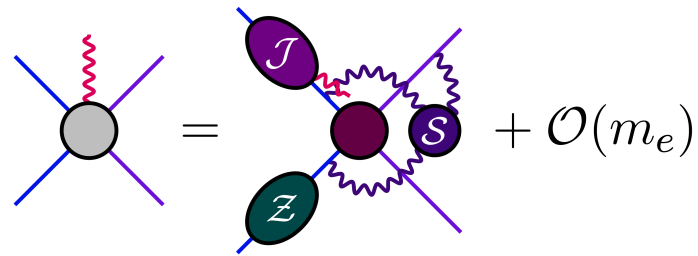
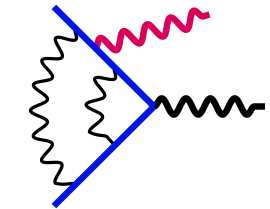


numerical instabilities
 \Rightarrow collinear subtraction? [Dittmaier et al. 08]

on our to-do list ?

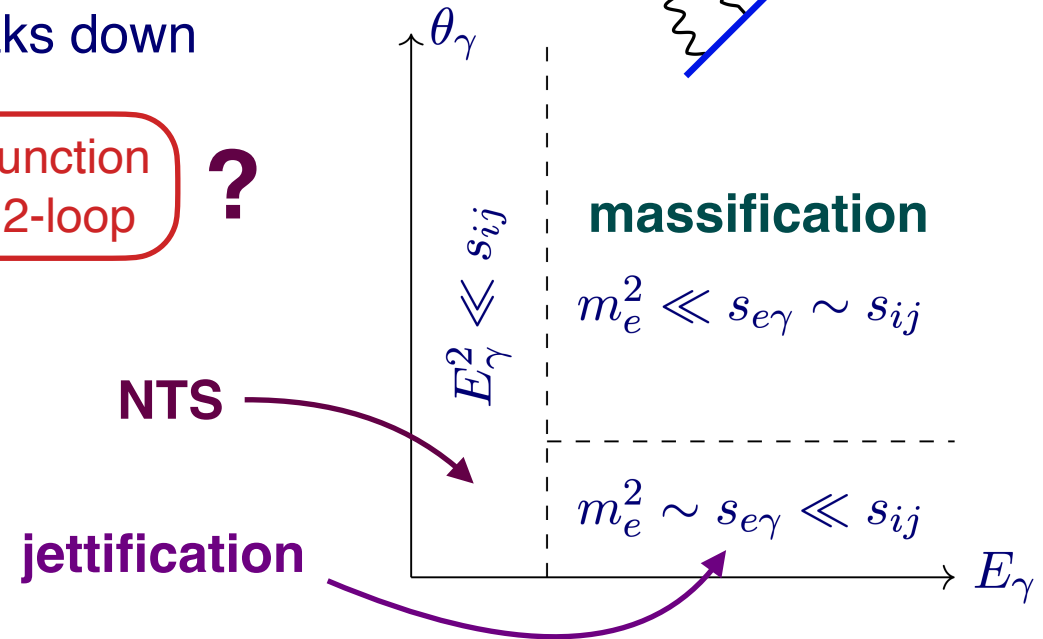
hard-collinear emission $\theta_\gamma \rightarrow 0$, i.e. $m_e^2 \sim s_{e\gamma}$ \sim KLOE-like SA scenario

massification for **virtual-virtual** corrections breaks down
 \Rightarrow **jettification** [Sara's talk]

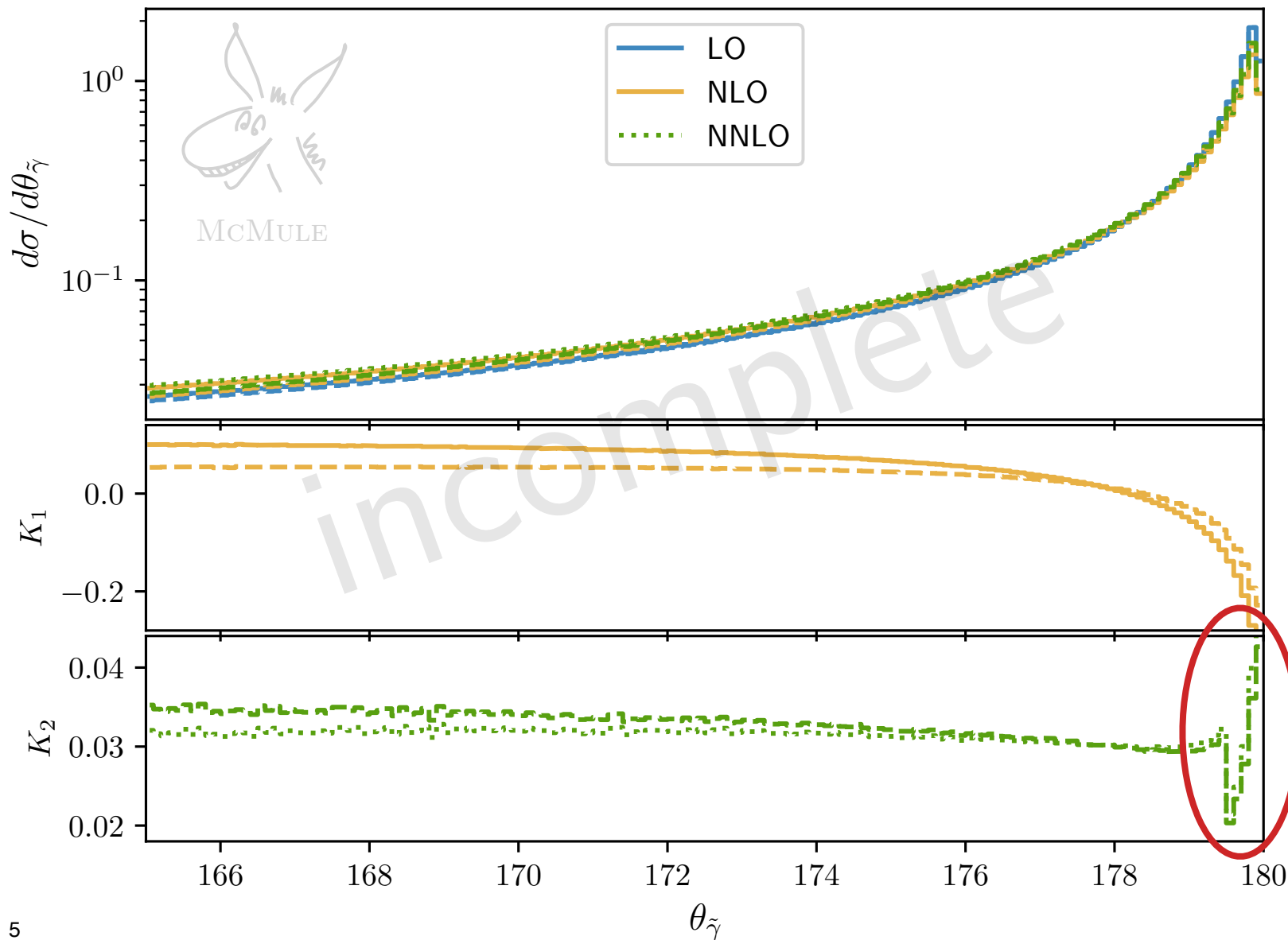


massive jet function unknown at 2-loop ?

... but no problems for \sim KLOE-like LA scenario

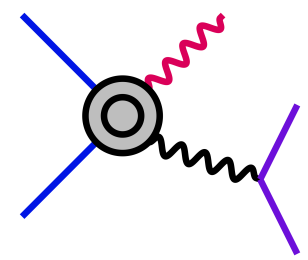


Results



KLOE-SA

- $\sqrt{s} = 1.02 \text{ GeV};$
- $50^\circ \leq \theta^\pm \leq 130^\circ;$
- $|p_z^\pm| > 90 \text{ MeV} \vee |p_\perp^\pm| > 160 \text{ MeV};$
- $\theta_{\tilde{\gamma}} \leq 15^\circ \vee \theta_{\tilde{\gamma}} \geq 165^\circ;$
- $0.35 \text{ GeV}^2 \leq M_{\mu\mu}^2 \leq 0.95 \text{ GeV}^2$



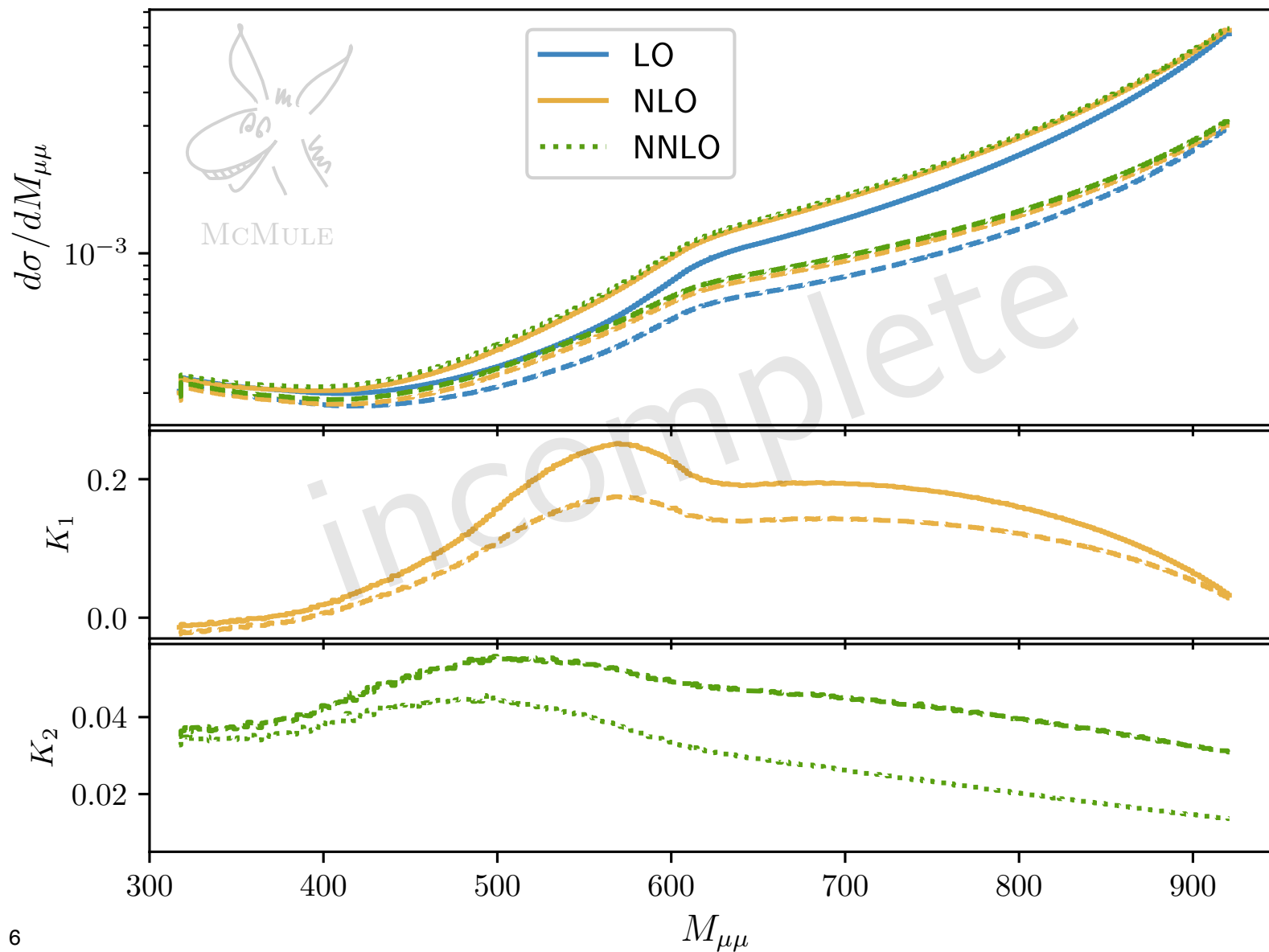
hard-collinear emission

cut $\theta_{\tilde{\gamma}} \leq 179.5^\circ$ for virtual-virtual corrections

⇒ error, ξ_c dependence (VV missing)

[Sara's talk]

Results



KLOE-LA

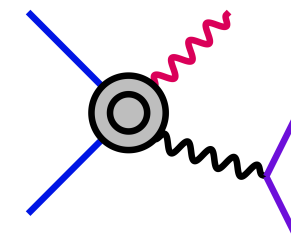
$$\sqrt{s} = 1.02 \text{ GeV};$$

$$50^\circ \leq \theta^\pm \leq 130^\circ;$$

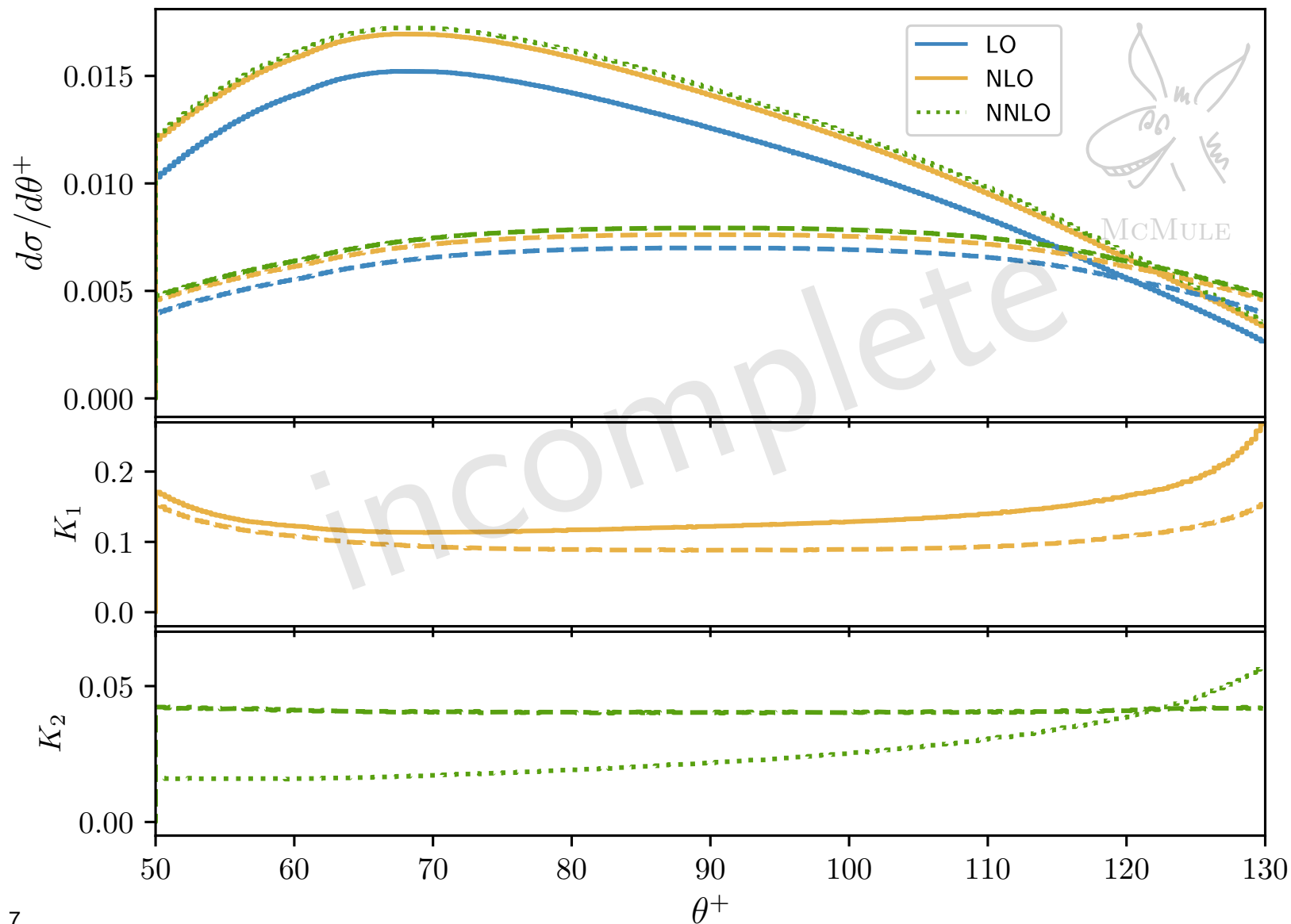
$$|p_z^\pm| > 90 \text{ MeV} \vee |p_\perp^\pm| > 160 \text{ MeV};$$

$$50^\circ \leq \theta_\gamma \leq 130^\circ \wedge E_\gamma > 20 \text{ MeV};$$

$$0.1 \text{ GeV}^2 \leq M_{\mu\mu}^2 \leq 0.85 \text{ GeV}^2$$



Results



KLOE-LA

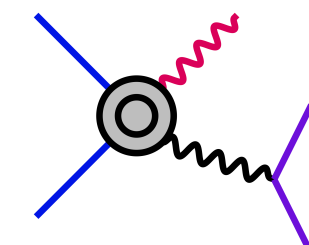
$$\sqrt{s} = 1.02 \text{ GeV};$$

$$50^\circ \leq \theta^\pm \leq 130^\circ;$$

$$|p_z^\pm| > 90 \text{ MeV} \vee |p_\perp^\pm| > 160 \text{ MeV};$$

$$50^\circ \leq \theta_\gamma \leq 130^\circ \wedge E_\gamma > 20 \text{ MeV};$$

$$0.1 \text{ GeV}^2 \leq M_{\mu\mu}^2 \leq 0.85 \text{ GeV}^2$$



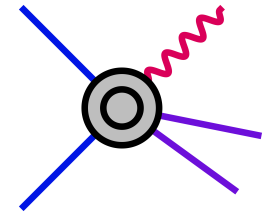
Next steps



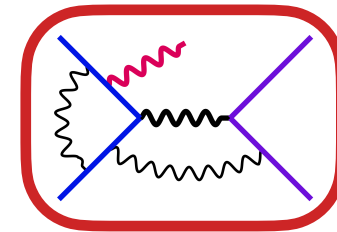
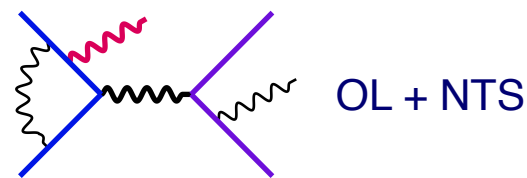
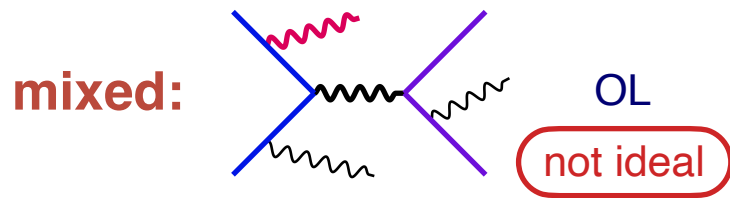
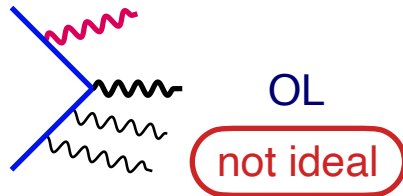
full $e^+e^- \rightarrow \mu^+\mu^-\gamma$ (ISC + FSC + mixed) with $m_e^2 \sim m_\mu^2 \ll s_{ij}$ up to NNLO

↪ bosonic corrections: no theoretical showstoppers

NLO solved (matrix elements with full mass dependence from OL)



current state @ NNLO: ISC ✓ ⇒ FSC ✓

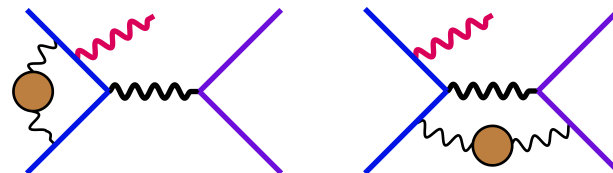


not yet included

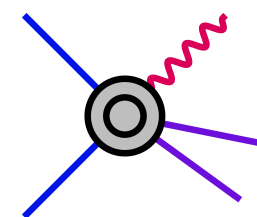
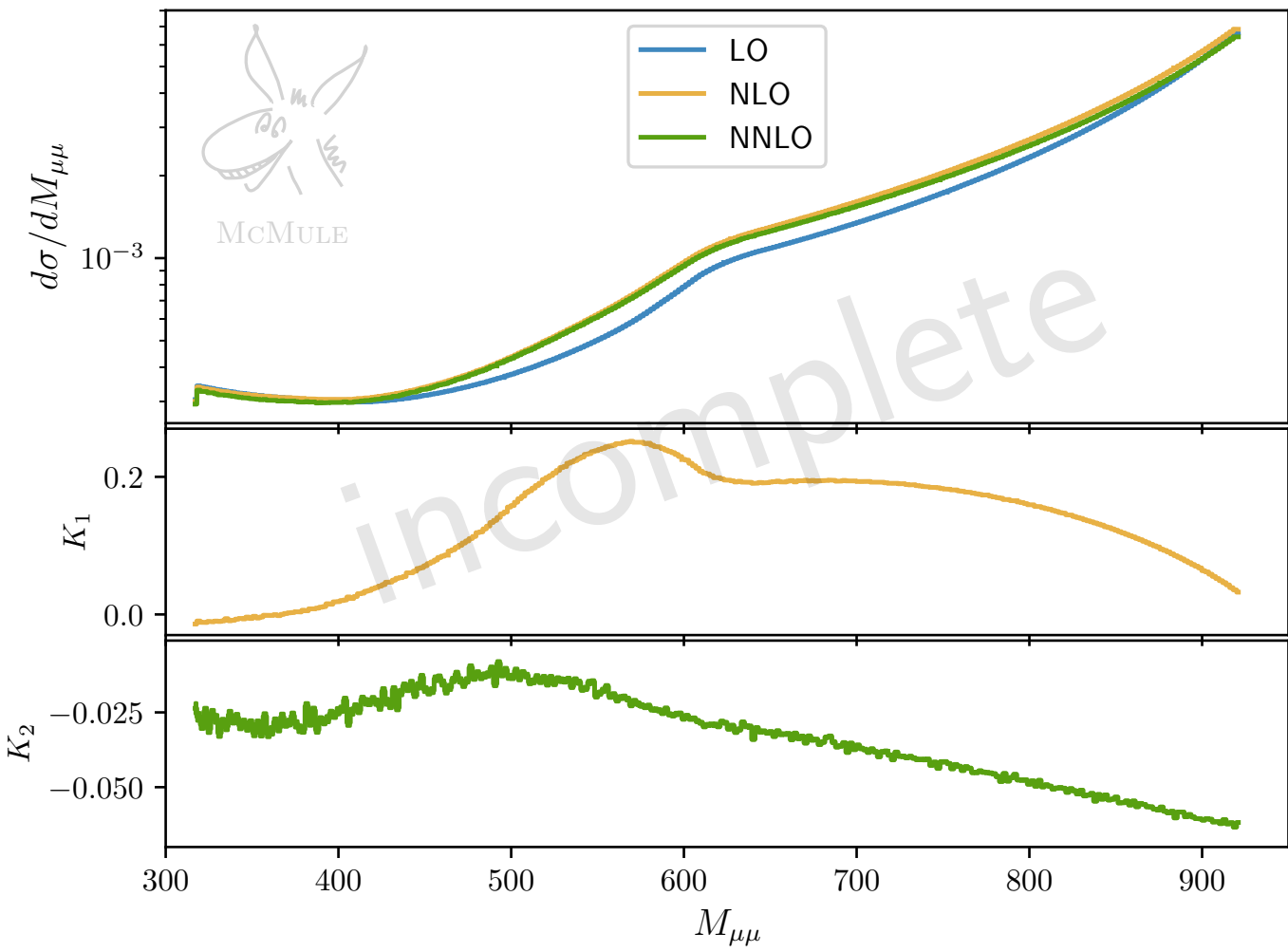
massification (m_e & m_μ)

amplitudes from
 $pp \rightarrow 2j + \gamma$
[Badger et al. 23]

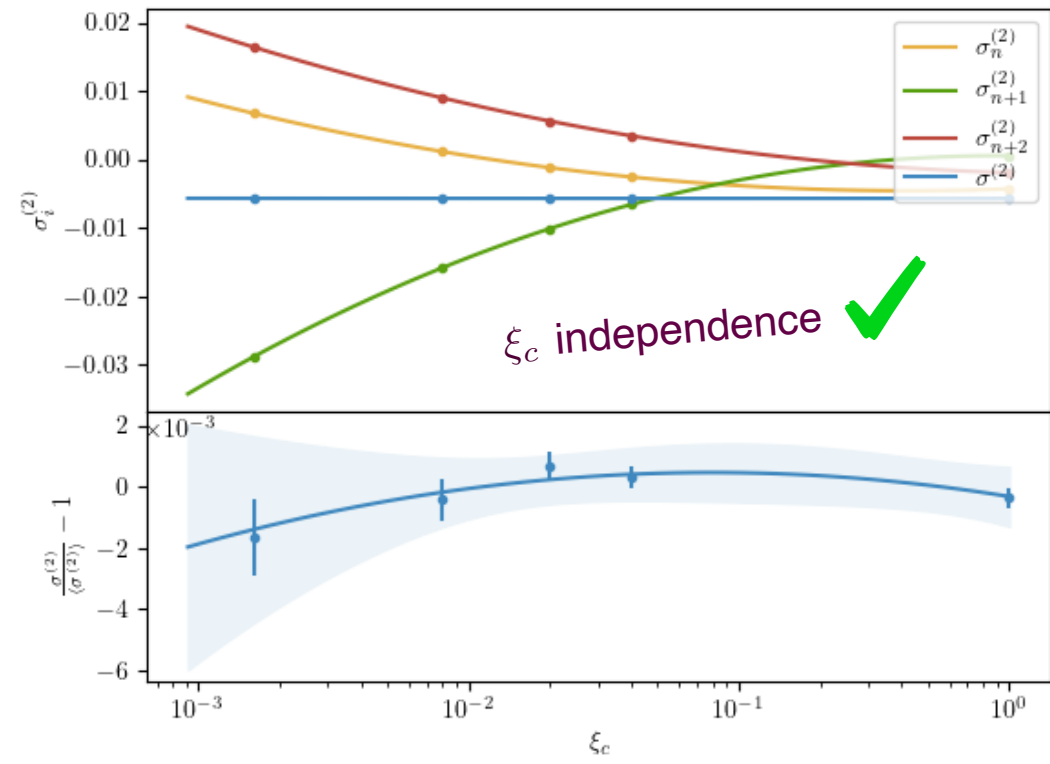
still missing: fermionic contributions



Results for KLOE-LA: full NNLO w/o virtual-virtual



check stability of full RR & RV

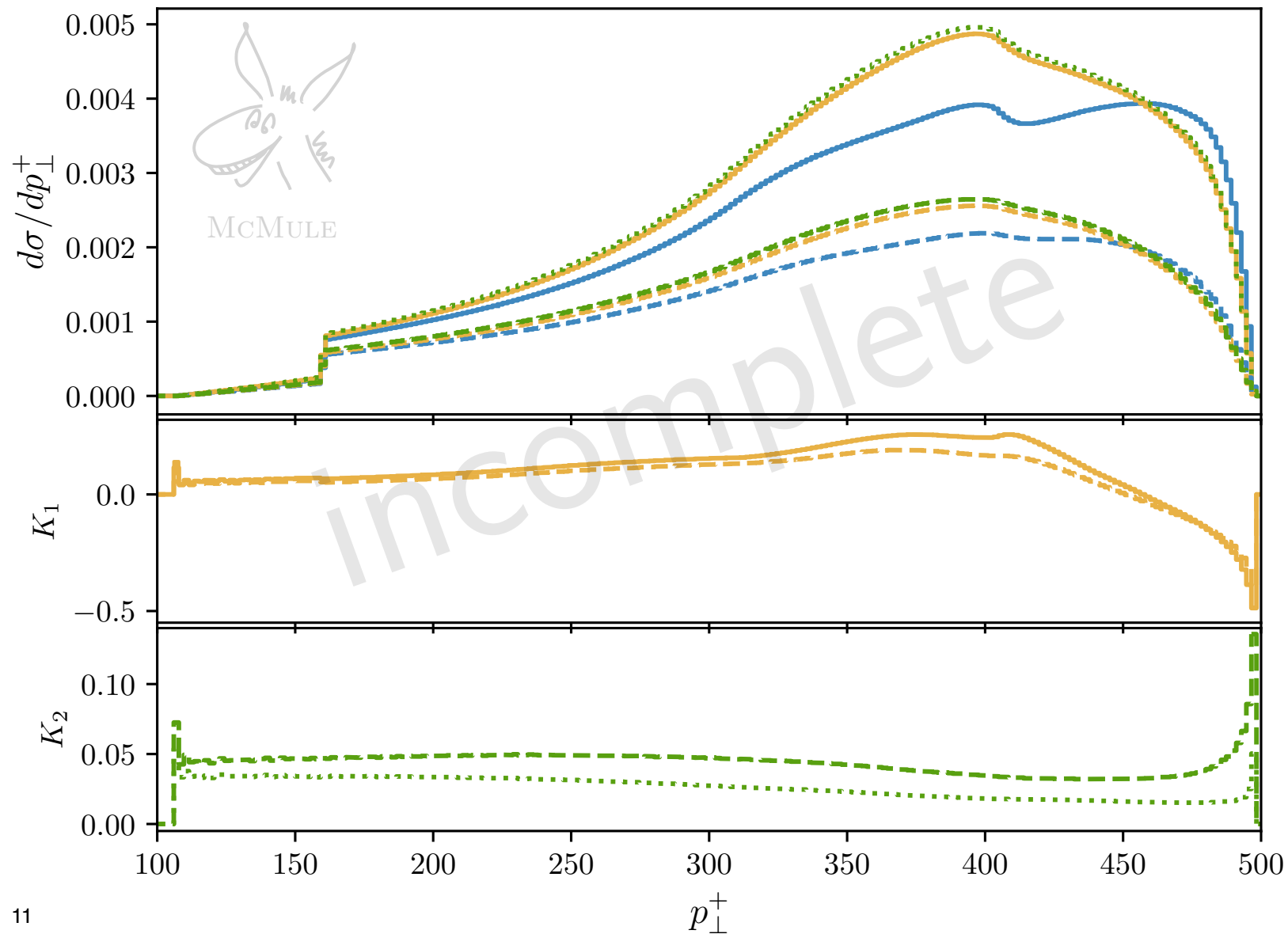


Conclusions



- ISC / FSC at NNLO under control for $m_{e/\mu}^2 \ll s_{ij}$ ✓
 - results surprisingly large (?) (analysis not yet completed)
 - full NNLO: first checks for RR & RV successful
 - full $ee \rightarrow \mu\mu\gamma$ at NNLO fairly doable in the limit $m_e^2 \sim m_\mu^2 \ll s_{ij}$
 - **However, ...**
 - numerical instabilities for BES- & B-like energies
 - massification of virtual-virtual corrections does not work for hard-collinear emission
- ⇒ needs to be investigated further

Appendix



KLOE-LA

$$\sqrt{s} = 1.02 \text{ GeV};$$

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$$50^\circ \leq \theta_\gamma \leq 130^\circ \wedge E_\gamma > 20 \text{ MeV};$$

$$0.1 \text{ GeV}^2 \leq M_{\mu\mu}^2 \leq 0.85 \text{ GeV}^2$$

