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IN KRAKÓW

Automation of (N)NLO Calculations with YFS Resummation

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How to treat QED Corrections?

Collinear Resummation

- Collinear logs are resummed with universal PDF $(P_T = 0)$
- Recently matched to NLO
- Combined with Parton Shower to generate photon emissions
- Beyond NLO becomes tricky

Jadach et.al, Z.Phys.C 49 (1991) 577-584,Europhys. Lett.17(1992) 123–128

S.Frixone et.al JHEP 03 (2020)

Soft Resummation

- Soft logs resummed to infinite orde using the YFS theorem
- Correct soft limit achieved for n photons
- Provides a robust scheme for the inclusion of real and virtual corrections at any order.
- Provides an exact treatment of multi photon phasespace

$$d\sigma(L, \hat{L}) = \alpha^k \sum_{n}^{\infty} \alpha^n \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \hat{\sigma}_{n,i,j} L^i \hat{L}^j$$

$$\hat{L} = \log\left(\frac{Q^2}{E_\gamma^2}\right) \qquad L = \log\left(\frac{Q^2}{m_e^2}\right)$$

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YFS Theorem

Yennie, Frautschi, and Suura showed how to reorder the entire perturbative series such that all IR divergences are resummed

It also provides an analytical treatment of the multi-photon phasespace

See talk by Z.Was

ANNALS OF PHYSICS: **13:** 379–452 (1961)

The Infrared Divergence Phenomena and High-Energy Processes*

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S. C. FRAUTSCHI‡

Department of Physics, University of California, Berkeley, California

AND

H. SUURA

Department of Physics, Nihon University, Tokyo, Japan

A general treatment of the infrared divergence problem in quantum electrodynamics is given. The main feature of this treatment is the separation of the infrared divergences as multiplicative factors, which are treated to all orders of perturbation theory, and the conversion of the residual perturbation expansion into one which has no infrared divergence, and hence no need for an infrared cutoff. In the infrared factors, which are exponential in form, the infrared divergences arising from the real and virtual photons cancel out in the usual way. These factors can then be expressed solely in terms of the momenta of the initial and final charged particles and an integral over the region of phase space available to the undetected photons; they do not depend upon the specific details of the interaction. Electron scattering from a static potential is treated in considerable detail, and several other examples are discussed briefly. As an important byproduct of the general treatment, it is found that when the infrared contributions are separated in a particular way, they dominate the radiative corrections at high energies and together with certain "magnetic terms" and vacuum polarization corrections seem to give all the contributions proportional to ln (E/m). All of these corrections can be easily estimated (in most cases) simply from a knowledge of the external momenta of the charged particles; this then provides a very powerful and accurate way of estimating radiative corrections to high-energy processes.

* Supported in part by the U. S. Atomic Energy Commission, Contract AT(11-1)-50.

† Various refinements were added to the manuscript (particularly in Appendix C) during the academic year 1960–1961 while this author was a National Science Foundation Senior Fellow visiting the University of Paris. He is grateful to Professor M. M. Lévy for the hospitality afforded by the Laboratoire de Physique Théorique et Hautes Énergies at Orsay.

‡ Supported by National Science Foundation Grant.

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YFS Master Equation

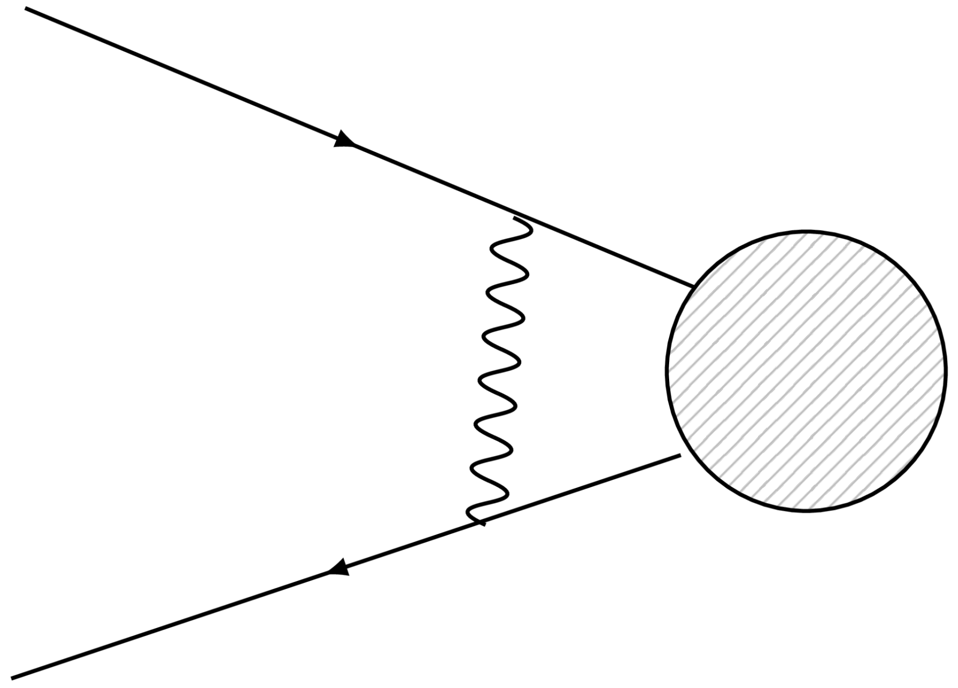
$$d\sigma = \sum_{n_\gamma=0}^{\infty} \frac{e^{Y(\Omega)}}{n_\gamma!} d\Phi_Q \left[\prod_{i=1}^{n_\gamma} d\Phi_i^\gamma \, \tilde{S}(k_i) \, \Theta(k_i, \Omega) \right] \left(\tilde{\beta}_0 + \sum_{j=1}^{n_\gamma} \frac{\tilde{\beta}_1(k_j)}{\tilde{s}(k_j)} + \sum_{\substack{j,k=1 \\ j<k}}^{n_\gamma} \frac{\tilde{\beta}_2(k_j, k_k)}{\tilde{s}(k_j)\tilde{s}(k_k)} + \cdots \right)$$

This expression contains **no approximations**.It does require any further matching. The accuracy is limited by how far you can calculate the betas

$$Y(\Omega) = \sum_{i<j} \mathscr{Re}\, B_{ij}(\Phi_n) + \tilde{B}_{ij}(\Phi_{n+1})$$

Virtual Emissions

Taking the soft limit allows us to factorise out amplitude



$$\mathscr{M}_0^1 = \alpha B M_0^0 + M_0^1$$

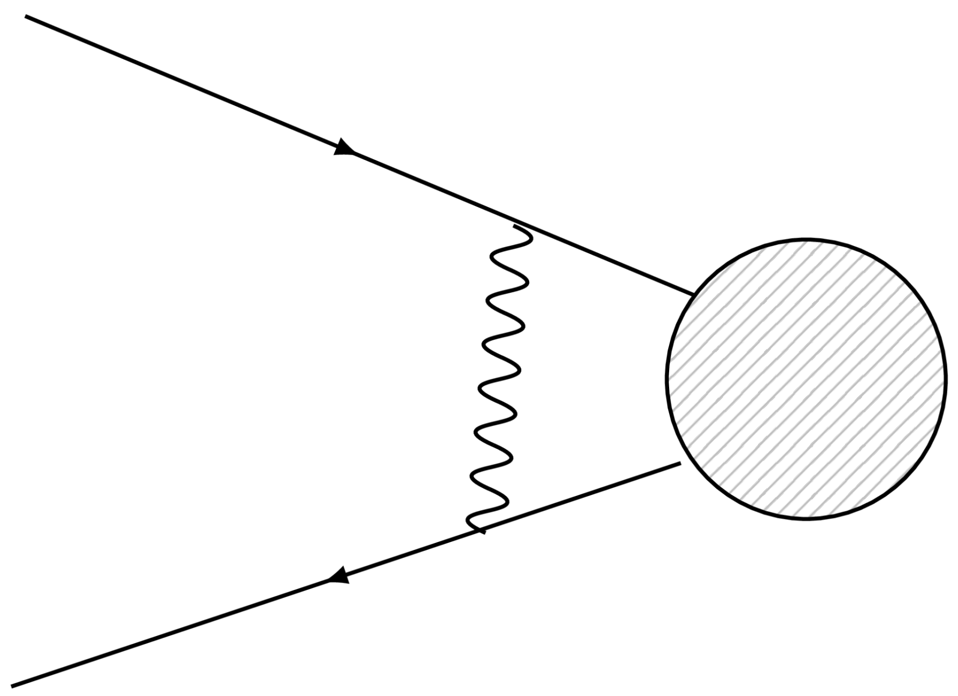
Full One-loop amplitude, including IR divergence

IR finite contribution

IR divergent contribution

Virtual Emissions

Taking the soft limit allows us to factorise out amplitude



$$\mathscr{M}_0^1 = \alpha B M_0^0 + M_0^1$$

$$\boxed{B_{ij} = -\frac{i}{8\pi^3} Z_i Z_j \theta_i \theta_j \int \frac{d^4k}{k^2} \left(\frac{2p_i\theta_i - k}{k^2 - 2(k\cdot p_i)\theta_i} + \frac{2p_j\theta_j + k}{k^2 + 2(k\cdot p_j)\theta_j} \right)^2}$$

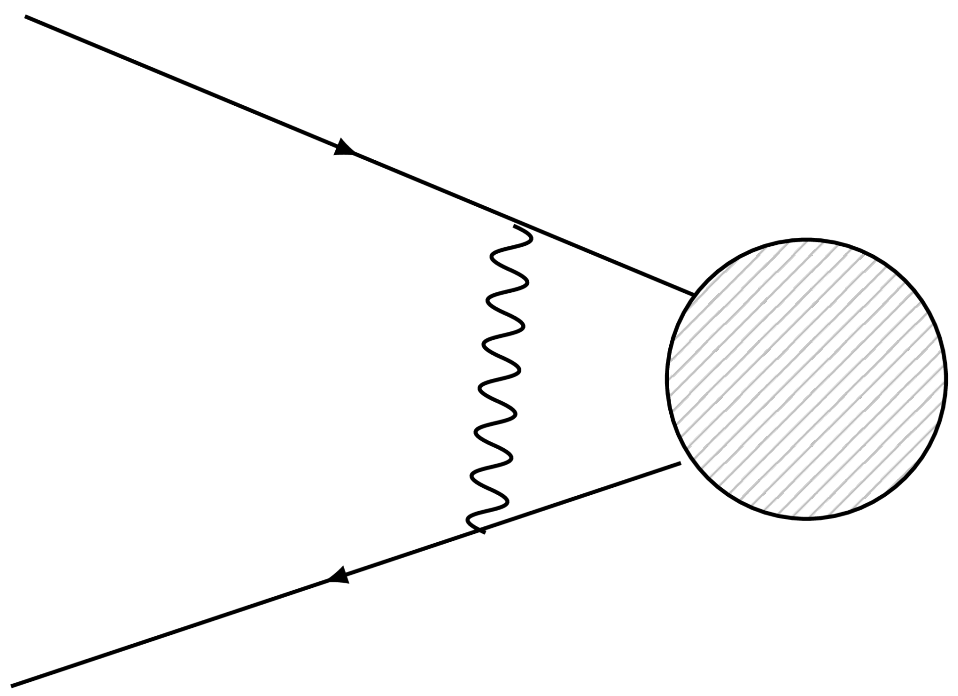
Universal Virtual

$Z_i =$ Particle Charge

$\theta_i = 1(-1)$ Incoming (Outgoing)

Virtual Emissions

This factorization continues at each order thanks to the abelian nature of QED



$$\mathscr{M}_0^0 = M_0^0$$

$$\mathscr{M}_0^1 = M_0^1 + \alpha B M_0^0$$

$$\mathscr{M}_0^2 = M_0^2 + \alpha B M_0^1 + \frac{(\alpha B)^2}{2!} M_0^0$$

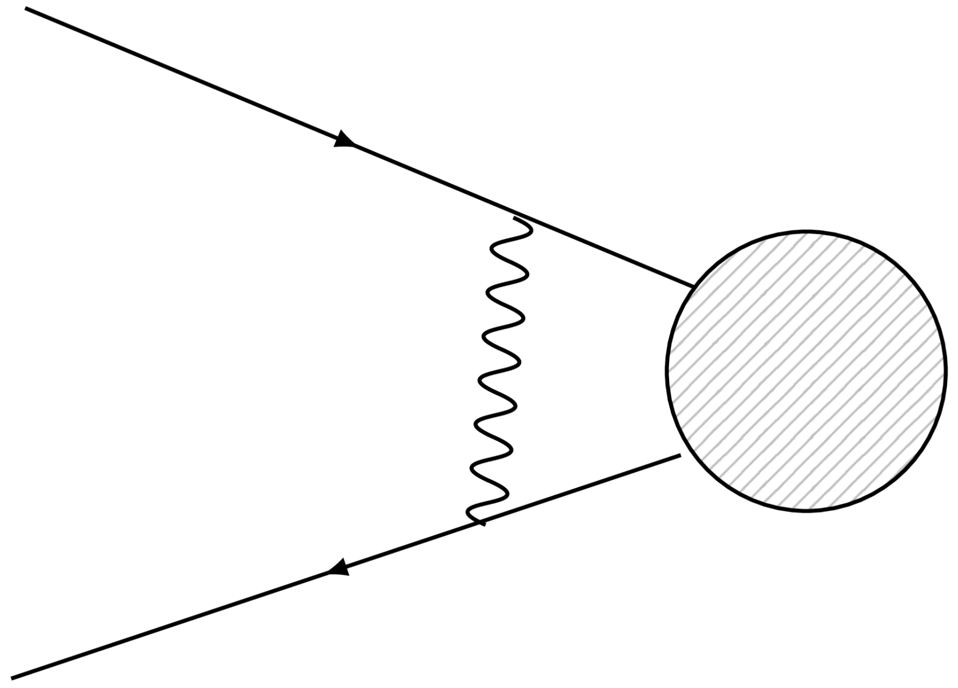
$$\Downarrow$$

$$\mathscr{M}_0^{\bar{n}_\gamma} = \sum_{r=0}^{\bar{n}_\gamma} M_0^{\bar{n}_\gamma - r} \frac{(\alpha B)^r}{r!}$$

Virtual Emissions

This factorization continues at each order

We can then sum to infinity and due to the abelian nature of QED we can generalise to arbitrary real emissions



$$\left| \sum_{\bar{n}_\gamma=0}^{\infty} \mathscr{M}_{n_\gamma}^{\bar{n}_\gamma+\frac{1}{2}n_\gamma} \right|^2 = e^{2\alpha B} \left| \sum_{\bar{n}_\gamma}^{\infty} M_{n_\gamma}^{\bar{n}_\gamma+\frac{1}{2}n_\gamma} \right|^2$$

$$B_{ij} = -\frac{i}{8\pi^3} Z_i Z_j \theta_i \theta_j \int \frac{d^4k}{k^2} \left(\frac{2p_i\theta_i - k}{k^2 - 2(k \cdot p_i)\theta_i} + \frac{2p_j\theta_j + k}{k^2 + 2(k \cdot p_j)\theta_j} \right)^2$$

Virtual Corrections

$$\tilde{\beta}_0^1\left(\Phi_n\right) = \mathscr{V}(\Phi_n) - \sum_{ij} \tilde{\mathscr{D}}_{ij}\left(\Phi_{ij} \otimes \Phi_n\right)$$

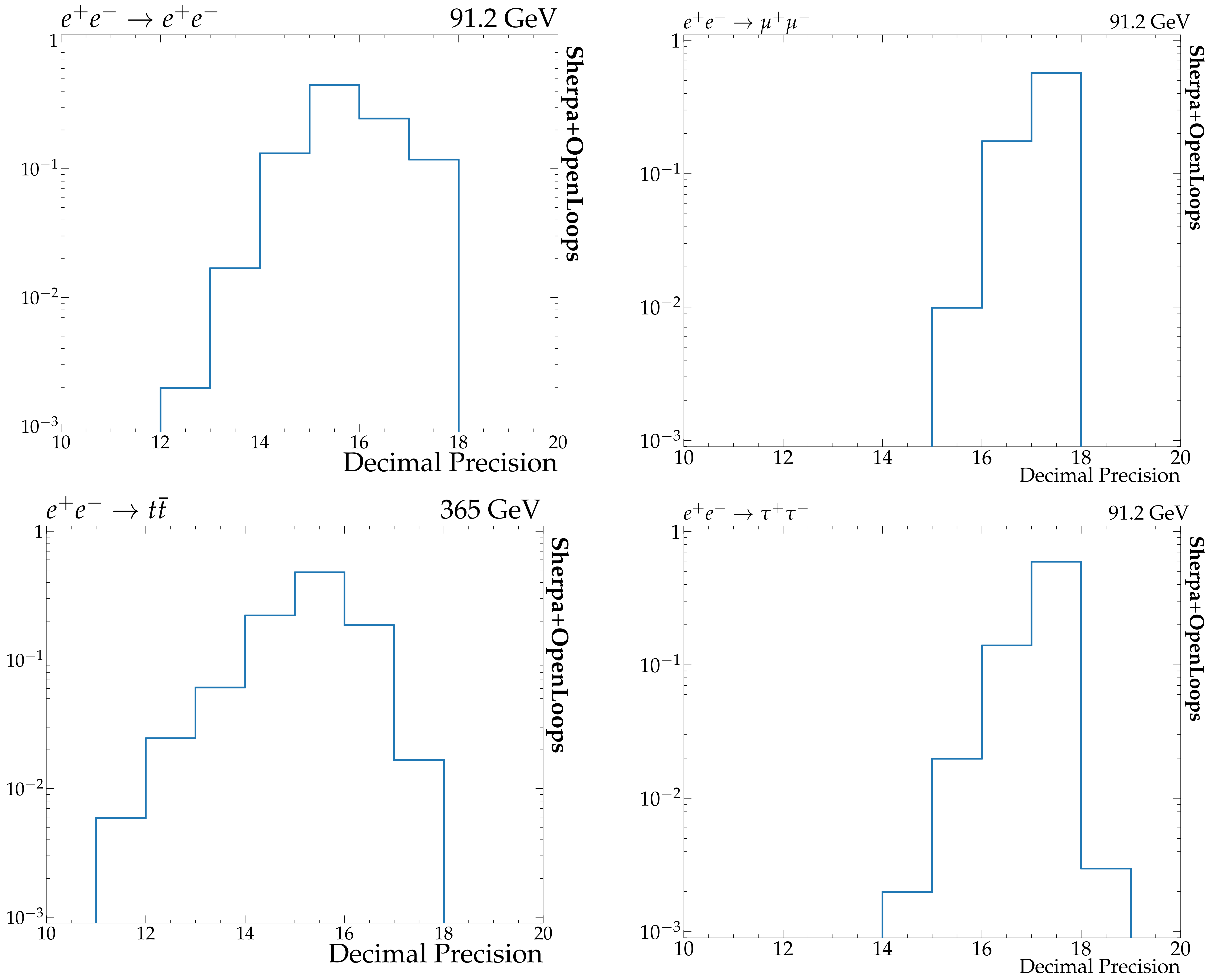
Sherpa automatically constructs the subtraction terms while external tools provide the **IR divergent** one-loop amplitude

These corrections contain no approximations e.g all masses are kept

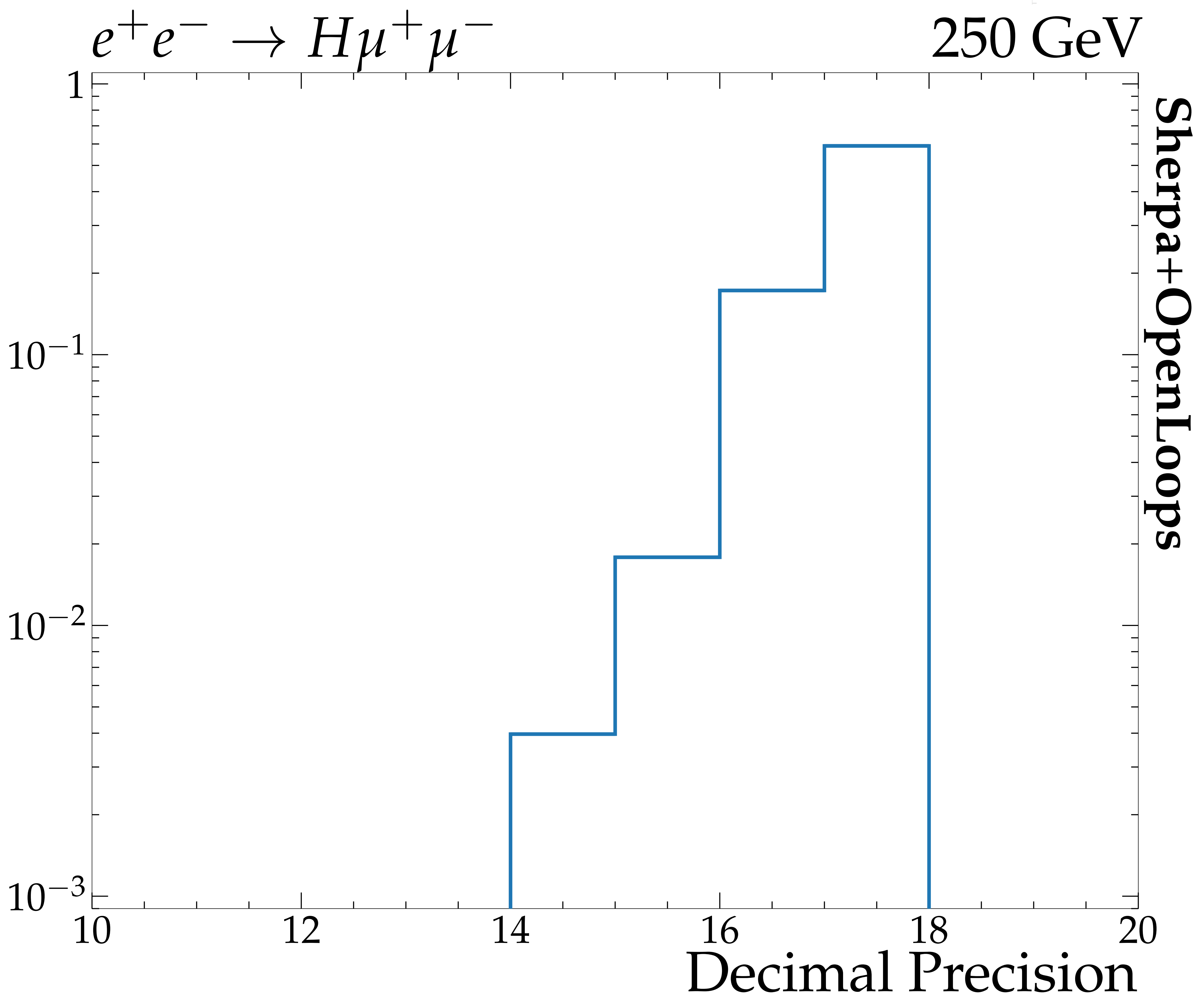
Results hold at low energy ~1GeV

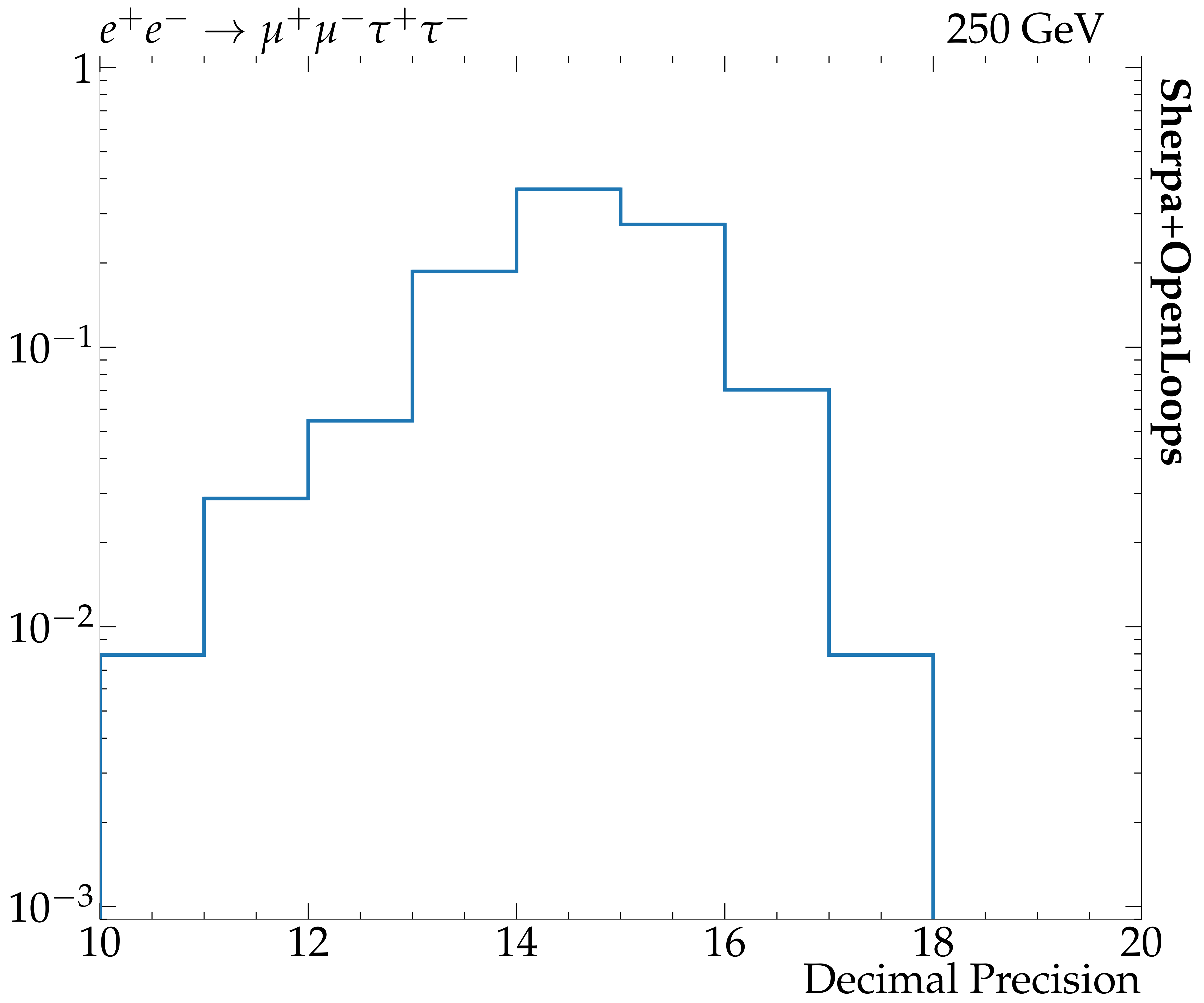
I can't calculate one-loop amplitudes but I can interface them

- Anonymous MC Author



Virtual Corrections





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Real Emissions

$$\frac{1}{2(2\pi)^3}\left|\sum_{\bar{n}_\gamma=0}^{\infty}\mathcal{M}_1^{\bar{n}_\gamma+\frac{1}{2}}\right|^2=\tilde{S}(k)\left|\sum_{\bar{n}_\gamma=0}^{\infty}M_0^{\bar{n}_\gamma}\right|^2+\sum_{\bar{n}_\gamma=0}^{\infty}\tilde{\beta}_1^{\bar{n}_\gamma+1}(k)$$

For real emissions, we consider the factorization at the amplitude squared level

$$\tilde{S}(k)=\sum_{i,j}\frac{\alpha}{4\pi^2}Z_iZ_j\theta_i\theta_j\left(\frac{p_i}{p_i\cdot k}-\frac{p_j}{p_j\cdot k}\right)^2$$

$Z_i =$ Charge

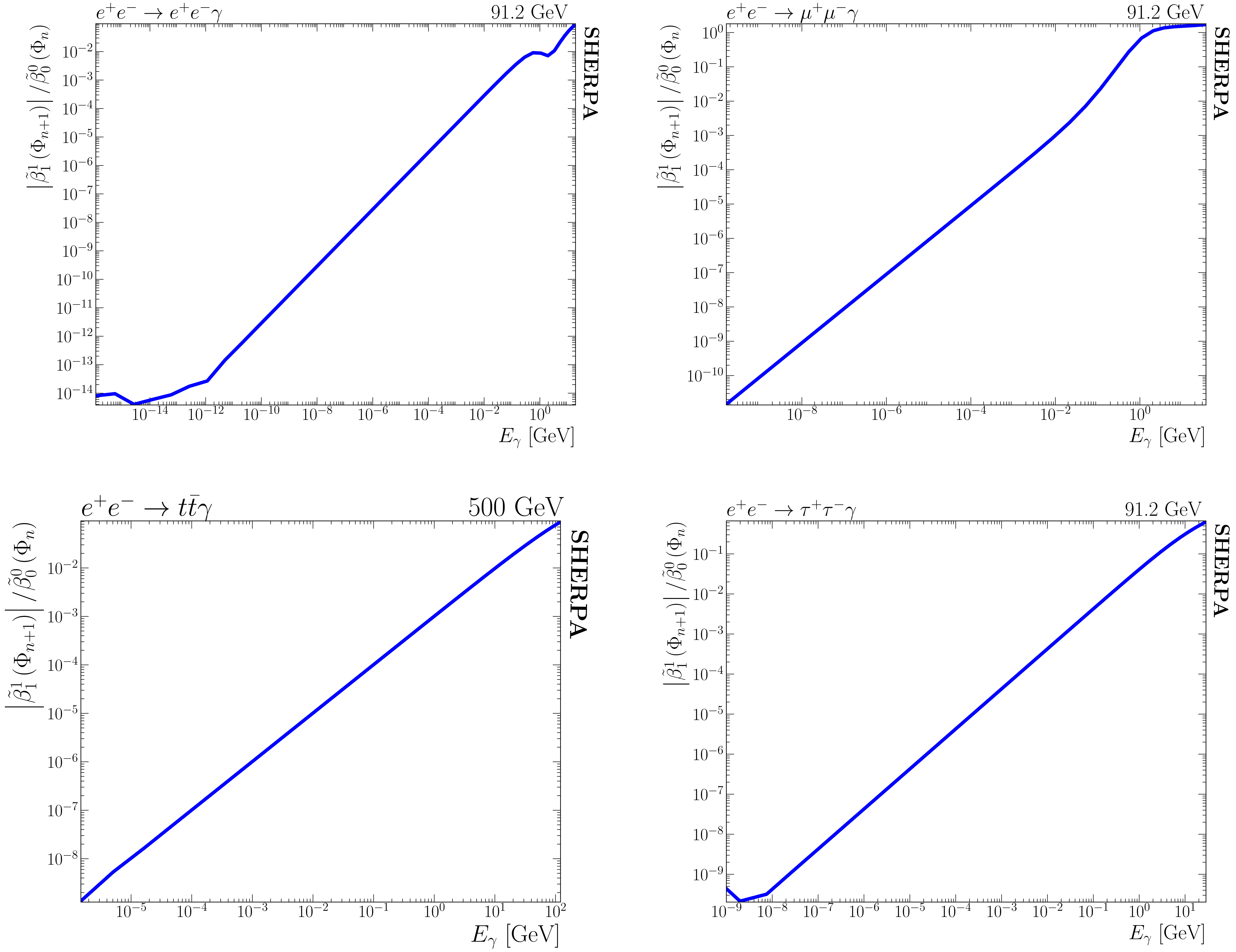
$\theta_i = 1(-1)$ Incoming (Outgoing)

Real Corrections

$$\tilde{\beta}_1^1\left(\Phi_{n+1}\right) = \mathscr{R}(\Phi_{n+1}) - \sum_{ij} \mathscr{D}_{ij}\left(\Phi_{ij+1} \otimes \Phi_n\right)$$

The real emissions are simple tree level amplitudes which can be calculated using standard methods in Sherpa

In the soft limit we see this contribution vanishes



Real-Virtual Corrections

$$\tilde{\beta}_1^2\left(\Phi_{n+1}\right) = \mathscr{R}\mathscr{V}(\Phi_{n+1}) - \sum_{ij} \mathscr{D}_{ij}^{(1)}\left(\Phi_{ij+1}\otimes\Phi_n\right).$$

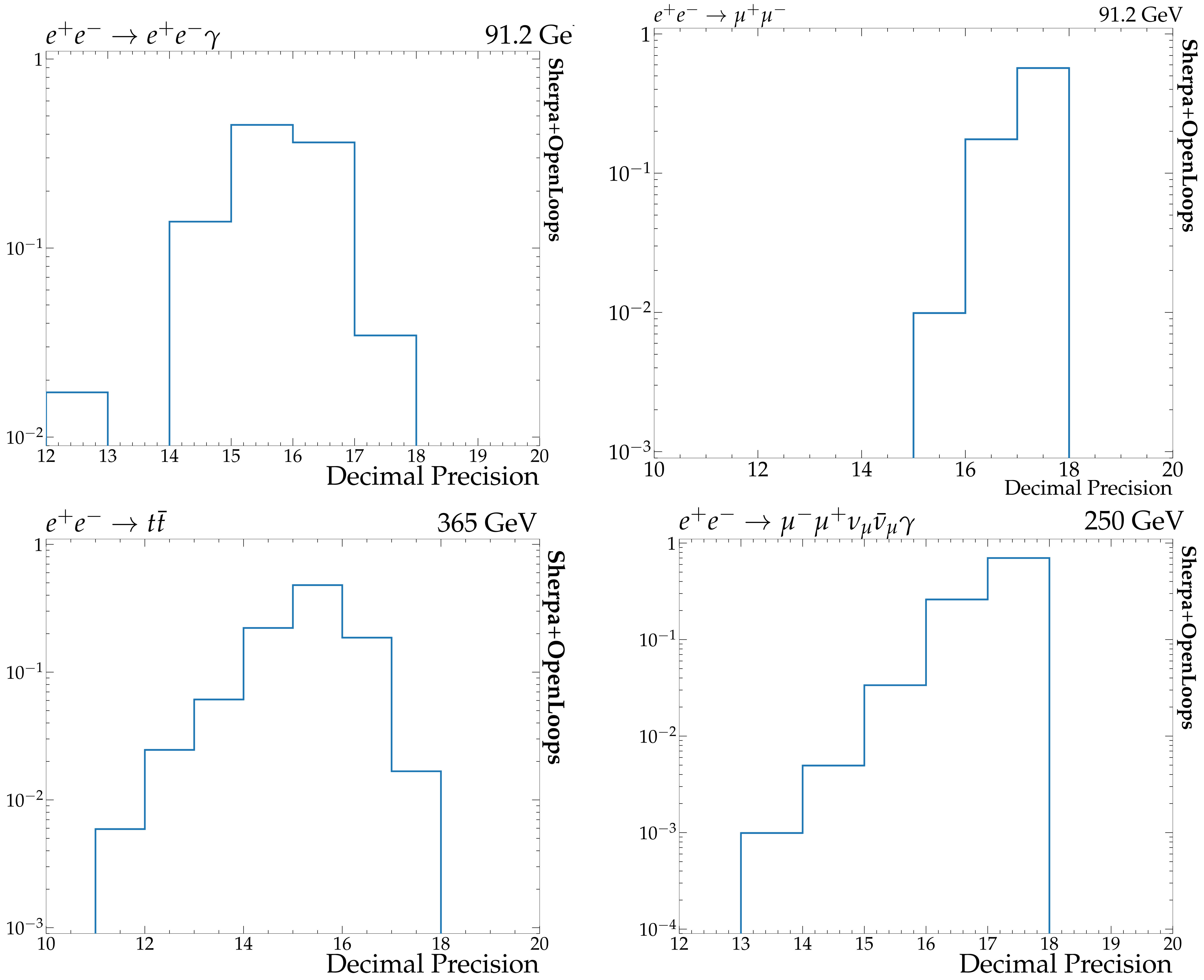
One-loop amplitudes again provided by external tool.

Sherpa again automatically constructs the subtraction term

These corrections contain no approximations e.g all masses are kept

I can't calculate one-loop amplitudes but I can interface them

- Anonymous MC Author



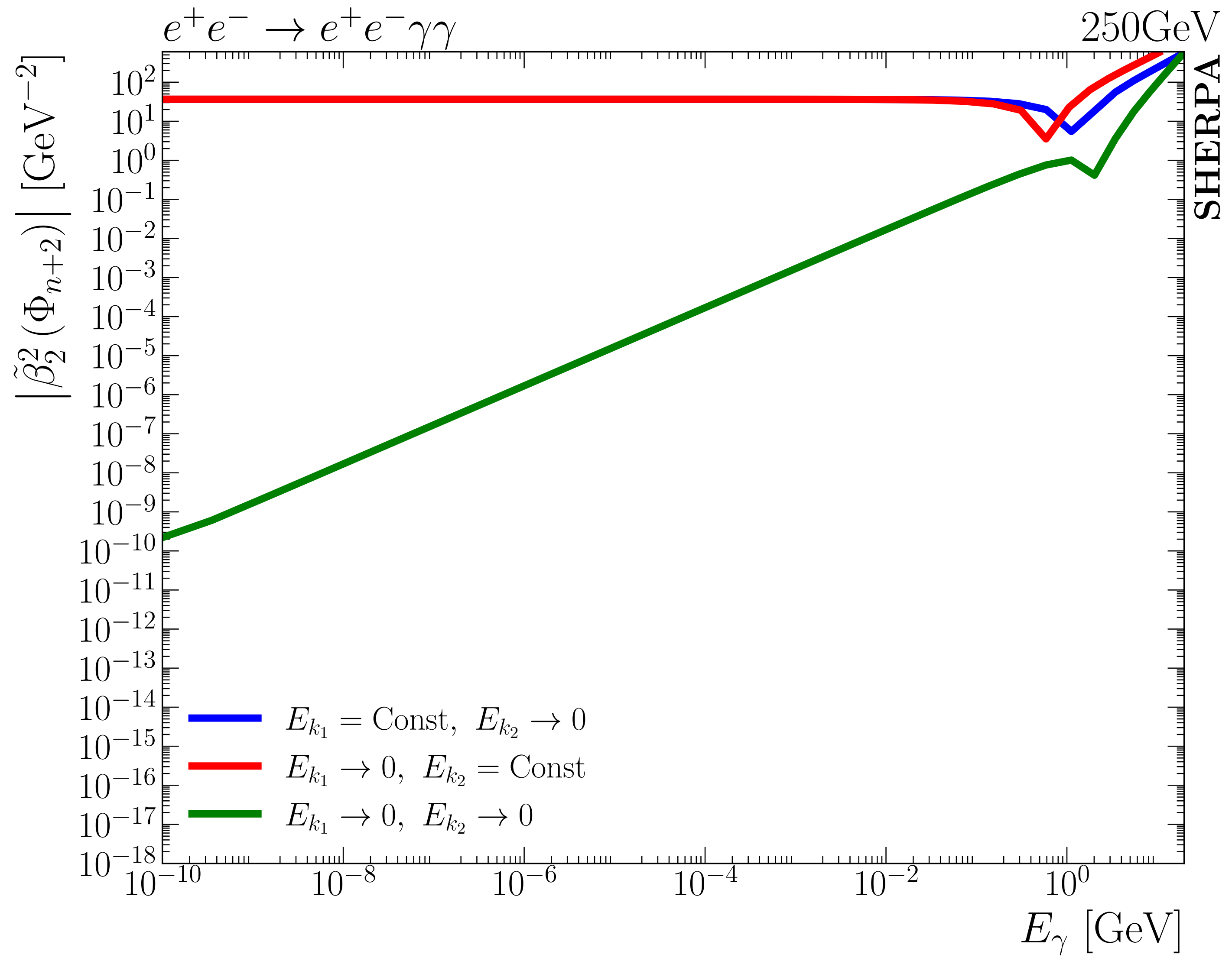
Double Real Corrections

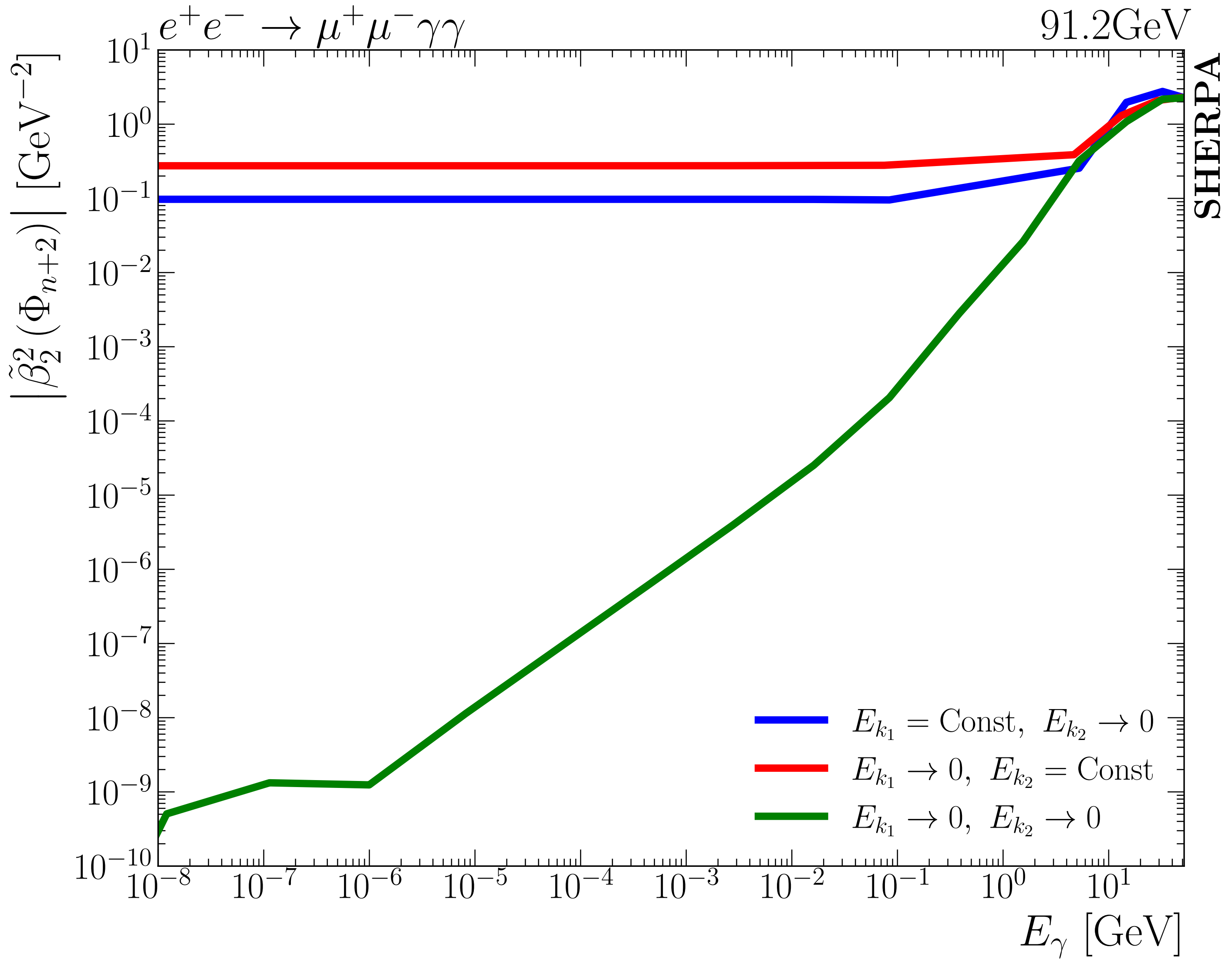
$$\tilde{\beta}_2^2\left(\Phi_{n+2}\right) = \mathcal{RR}(\Phi_{n+2})$$

$$-\tilde{S}(k_1)\tilde{\beta}_1^1\left(\Phi_{n+1};k_2\right)$$

$$-\tilde{S}(k_2)\tilde{\beta}_1^1\left(\Phi_{n+1};k_1\right)$$

$$-\tilde{S}(k_1)\tilde{S}(k_2)\tilde{\beta}_0^0\left(\Phi_n\right)$$





By far the most complicated subtraction term

Double Virtual Corrections

Unfortunately, there is not automated tool for the calculation of double virtual correction.

GRIFFIN: A C++ library for EW radiative corrections [2211.16272]
Developed by A. Freitas and L.Chen

- Modern version of LEP era EW tools such as DIZET (used by KKMC)
- Only for $f\bar{f} \to f'\bar{f}'$
- NNLO accurate at the Z pole

For low-energy e+e- these corrections could be provided by dizet
In the meantime we take the simple collinear approximation of the two-loop corrections

Pion Production

We now support $\pi^+\pi^-$ production.

The higher order corrections are under construction but the resummation can be applied

CODE	mmg	ppg	Comments (matrix element, FSC)
Sherpa	NLO+YFS	NLO+YFS	FxsQED, GVMD,FsQED

FsQED and FxsQED feasible within YFS framework, unsure of the timeline (manpower)

For more details on Sherpa => **L.Flower's talk**