Low Energy for KKMC and associated projects - Systematic Ambiguities what is to do

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- **(1)** KKMC for $e^+e^- \to l\bar{l}n\gamma, l = \mu, \tau, \nu, q$ (for τ's with decays) was developed over several decades, starting from low energies, but since long focus mostly on high energy applications.
- **(2)** Program is useful for low energy applications. Some aspects, technical and theoretical are not developed to the point, mostly because of man power issues.
- **(3)** Let me present basic aspects first ...
- **(4)** ...and new applications later.
- **(5)** Long term man power perspective looks somehow better these days.
- **(6)** **Talk plan:** (i) Basic construction elements. (ii) Tests and low energies limitations. (iii) Versions and distributions (iv) New applications.

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Basic construction elements

- Separation into τ production and decay and spin algorithm.

- QED exclusive exponentiation. Amplitudes are constructed from parts. Eikonal part, correction obtained from first order amplitudes, correction obtained from second order amplitude ... etc. That required lots of work.

- Separating out: bremsstrahlung part of QED. Separating from the rest short scale interaction part: electroweak effects, at low energies that means $\alpha_{QED}(s)$ valid at first order, analytic and anti analytic properties of field theory results.

- Separating out New Physics effects, including interfering ones.

- Phase space parametrization, crude distribution, all possible because of conformal symmetry.

- Hopefully Alan presented some of these points.

Basic construction elements

Formalism for $\tau^+\tau^-$

- Because narrow τ width approximation can be obviously used for phase space, cross-section for the process $f\bar{f} \to \tau^+\tau^- Y; \tau^+ \to X^+\bar{\nu}; \tau^- \to X^-\nu$ reads:

$$d\sigma = \sum_{spin} |\mathcal{M}|^2 d\Omega = \sum_{spin} |\mathcal{M}|^2 d\Omega_{prod}\, d\Omega_{\tau^+}\, d\Omega_{\tau^-}$$

- This formalism is fine, but, e.g. for 20 τ decay channels we would have 400 distinct processes. Also picture of production and decay are mixed.

- Below only τ spin indices are explicitly written:

$$\mathcal{M} = \sum_{\lambda_1\lambda_2=1}^{2} \mathcal{M}^{prod}_{\lambda_1\lambda_2}\, \mathcal{M}^{\tau^+}_{\lambda_1} \mathcal{M}^{\tau^-}_{\lambda_2}$$

Basic construction elements

- Cross section can be rewritten into **core formula of spin algorithms**

$$d\sigma = \Big(\sum_{spin} |\mathcal{M}^{prod}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^+}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^-}|^2\Big) wt\ d\Omega_{prod}\ d\Omega_{\tau^+}\ d\Omega_{\tau^-}$$

- where

$$wt = \Big(\sum_{i,j=0,3} R_{ij} h^i h^j\Big)$$

$$R_{00} = 1, \quad < wt >= 1, \quad 0 \leq wt \leq 4.$$

R_{ij} can be calculated from $\mathcal{M}_{\lambda_1\lambda_2}$ by contraction with Pauli σ^i matrices and similarly h^i, h^j respectively from $\mathcal{M}^{\tau^+}$ and $\mathcal{M}^{\tau^-}$.

- Bell inequalities tell us that it is impossible to rewrite wt in the following form

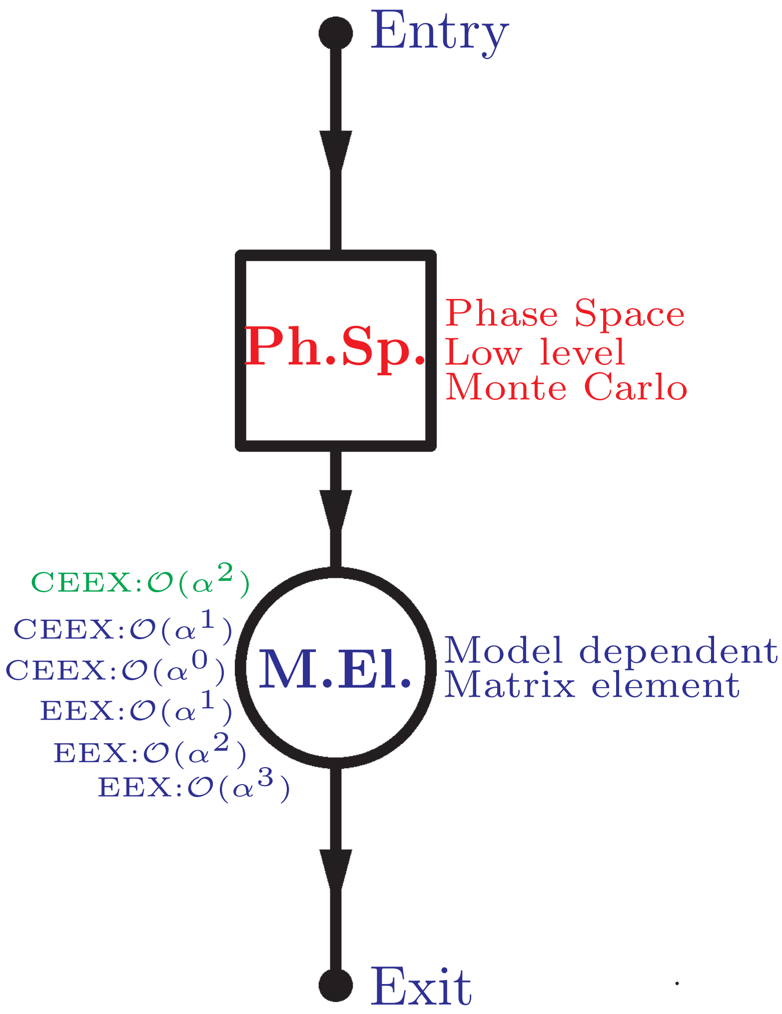
$$wt \neq \Big(\sum_{i,j=0,3} R^A_i h^i\Big)\Big(\sum_{i,j=0,3} R^B_j h^j\Big)$$

that means it is impossible to generate first τ^+ and τ^- first in some given ‘ quantum state’ and later perform separately decays of τ^+ and τ^-

\- Approx. only! **NO** each τ `spin` in `HepMC`. Fatal: anom. dipole mom. pheno.

Basic construction elements

Textbook principle “matrix element $\times$ full phase space” useful

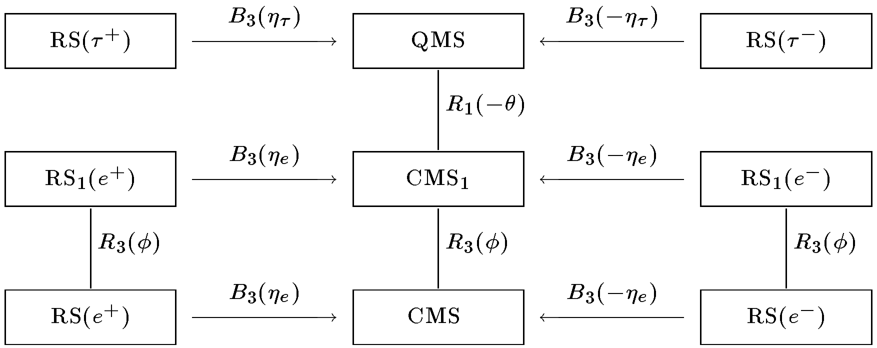


- Phase-space Monte Carlo module producing “raw events”.
- Library of models for provides input for “model weight”
- **Useful for any application, not only τ production/decay.**
- Ratios of matrix elements squared define weight (probability) that event could be of model B if generated with mode A.
- Convenient for Machine Learning too.
- No compromises on precision are required.

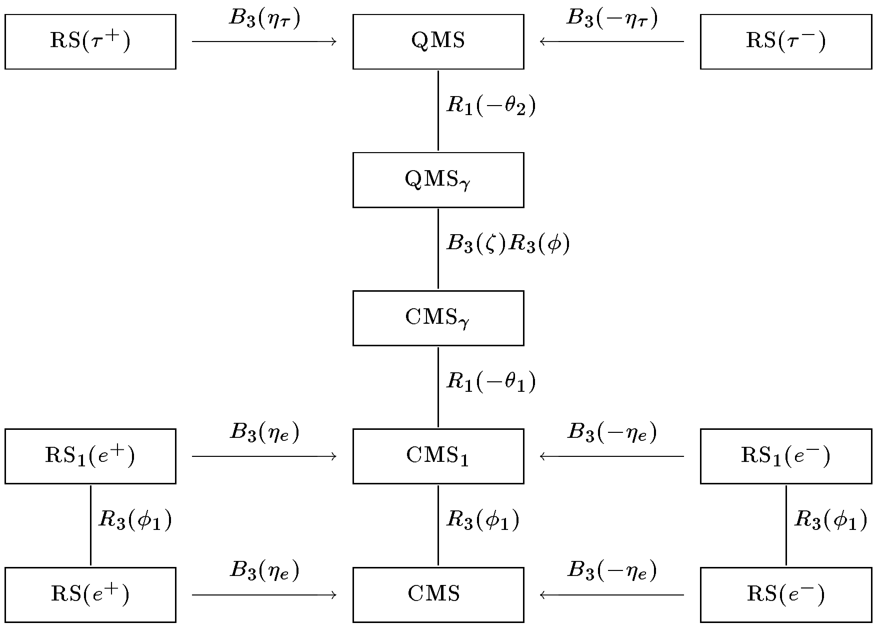
Basic construction elements

Ref. frames for spin; production, decay. Geometry of QED amplitudes optimized

Figure 2



(2a)



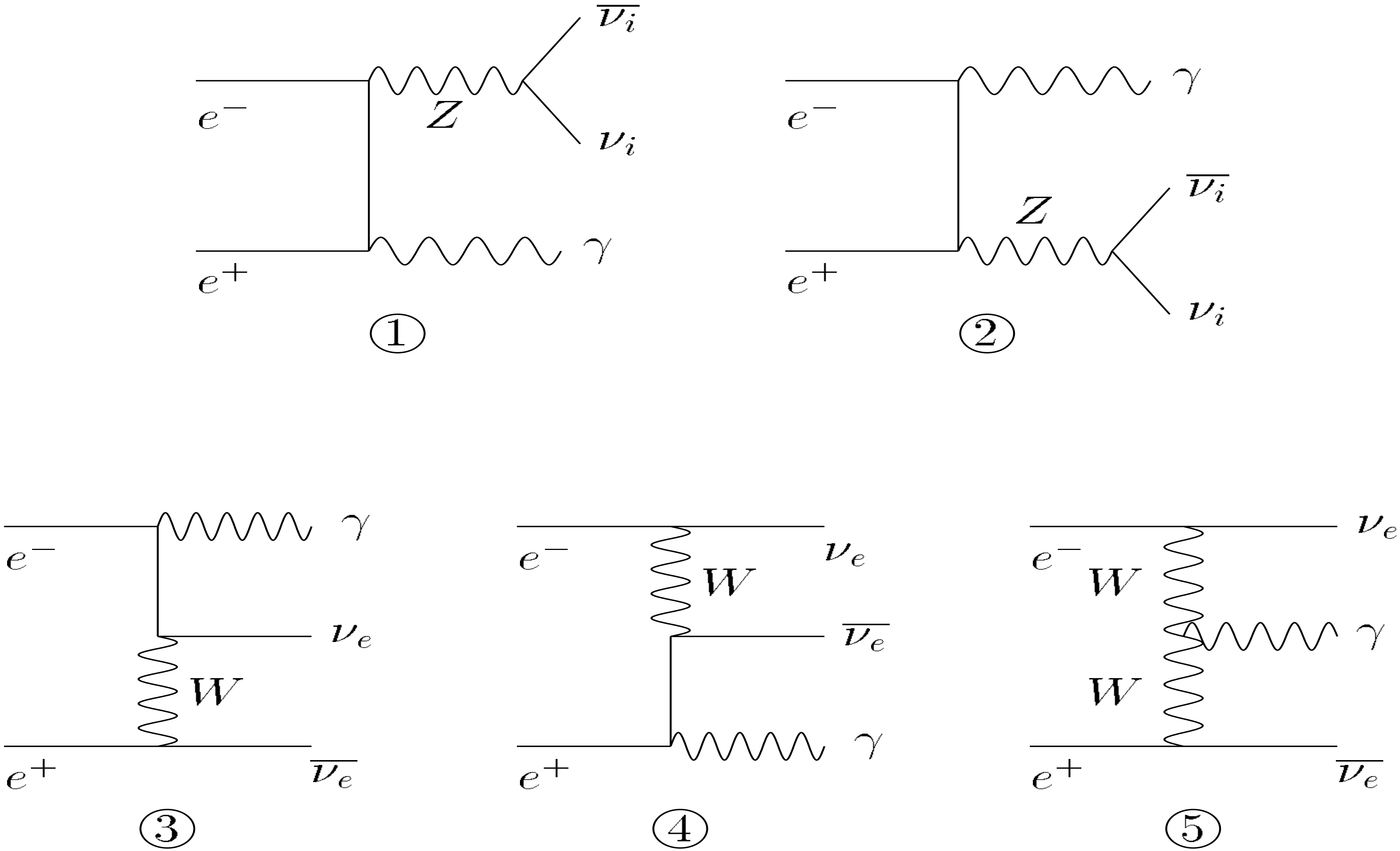
(2b)

Basic construction elements

- General idea: to identify in amplitudes, with the help of gauge invariance structures responsible later for phase-space enhancements: collinear-soft etc. This is fundamental, specially from the point of view of Monte Carlo algorithm construction.
- Discussions with **Shimizu-san** were important.
- Z. Was *Gauge invariance, infrared / collinear singularities and tree level matrix element for e+ e- —> nu(e) anti-nu(e) gamma gamma* Eur.Phys.J. C44 (2005) 489,
- A. van Hameren, Z. Was, *Gauge invariant sub-structures of tree-level double-emission exact QCD spin amplitudes*, Eur.Phys.J. C61 (2009) 33
- Also in this case algebraic manipulation methods were providing the reference calculations, necessary to cross check results.
- I was not able to find patterns automatically, but algebraic progams were essential for checks.
- Only some of the patterns appear naturally. Feynman diagrams 1 and 2 combined (next slide) are the complete amplitude for $\nu_\mu \bar{\nu}_\mu$ production.

Basic construction elements

Figure 1: *The Feynman diagrams for* $e^+e^- \to \bar{\nu}_e \nu_e \gamma$.



Basic construction elements

- The first two diagrams represent initial state QED bremsstrahlun amplitudes for $\nu_\mu \bar{\nu}_\mu$ pair production. It can be divided into parts, corresponding to β_0, β_1 of Yennie-Frautshi-Suura exponentiation.
- Can separation be expanded to other cases, to higher orders, to terms of different singularities/enhancements?
- The answer seem to be yes, provided we stay in QED regime
- But it comes only after lots of effort.
- Many of NN-calculations do not respect (or can not respect) such separation into parts.
- It is most important for real emissions: algorithms design. Virtual corrections: broad aspect of separation into parts.
- The case is **testing limits** of QED YFS reorganization of perturbation expansion.
- Holds at second order too, but then internal charged scalar lines contribute. Is it QED?
- Virtual mixed virtual corrections were challenging too. But expansion around contact interaction for t-channel W's worked fine.
- Important: red part of the second page equation clearly identifiable. Essential feature used for algorithm design.

Basic construction elements

$$
\begin{aligned}
\mathcal{M}_{1\{I\}} \left({}^{p\,k_1}_{\lambda\,\sigma_1} \right) &= \mathcal{M}^0 + \mathcal{M}^1 + \mathcal{M}^2 + \mathcal{M}^3 \\
\mathcal{M}^0 &= eQ_e\ \bar{v}(p_b,\lambda_b)\ \mathbf{M}^{bd}_{\{I\}}\ \frac{\not{p}_a + m - \not{k}_1}{-2k_1p_a}\ \not{\epsilon}^{\star}_{\sigma_1}(k_1)\ u(p_a,\lambda_a) \\
&+ eQ_e\ \bar{v}(p_b,\lambda_b)\ \not{\epsilon}^{\star}_{\sigma_1}(k_1)\ \frac{-\not{p}_b + m + \not{k}_1}{-2k_1p_b}\ \mathbf{M}^{ac}_{\{I\}}\ u(p_a,\lambda_a) \\
\mathcal{M}^1 &= \mathcal{M}^{1'} + \mathcal{M}^{1''} \\
\mathcal{M}^{1'} &= +e\ \bar{v}(p_b,\lambda_b)\ \ \mathbf{M}^{bd,ac}_{\{I\}}\ u(p_a,\lambda_a)\epsilon^{\star}_{\sigma_1}(k_1)\cdot(p_c-p_a)\frac{1}{t_a-M_W^2}\,\frac{1}{t_b-M_W^2}, \\
\mathcal{M}^{1''} &= +e\ \bar{v}(p_b,\lambda_b)\ \ \mathbf{M}^{bd,ac}_{\{I\}}\ u(p_a,\lambda_a)\epsilon^{\star}_{\sigma_1}(k_1)\cdot(p_b-p_d)\frac{1}{t_a-M_W^2}\,\frac{1}{t_b-M_W^2}, \\
\mathcal{M}^2 &= +e\ \bar{v}(p_b,\lambda_b)g^{We\nu}_{\lambda_b,\lambda_d}\ \not{\epsilon}^{\star}_{\sigma_1}(k_1)\ \ v(p_d,\lambda_d)\bar{u}(p_c,\lambda_c)g^{We\nu}_{\lambda_c,\lambda_a}\ \not{k}_1\ \ u(p_a,\lambda_a)\frac{1}{t_a-M_W^2}\,\frac{1}{t_b-M_W^2} \\
\mathcal{M}^3 &= -e\ \bar{v}(p_b,\lambda_b)g^{We\nu}_{\lambda_b,\lambda_d}\ \not{k}_1\ \ v(p_d,\lambda_d)\bar{u}(p_c,\lambda_c)g^{We\nu}_{\lambda_c,\lambda_a}\ \not{\epsilon}^{\star}_{\sigma_1}(k_1)\ \ u(p_a,\lambda_a)\frac{1}{t_a-M_W^2}\,\frac{1}{t_b-M_W^2},
\end{aligned}
\tag{1}
$$

- Complete spin amplitude for $e^+e^- \to \bar{\nu}_e\nu_e\gamma$ separated into *six* individually QED gauge invariant parts. Easy to check, replace photon polarization with its four-momentum. Each of the obtained parts has well defined physical interpretation. Eikonal universal infrared singular is in red. We touch essentials of pert. expansion reordering.
- It is also easy to verify that the gauge invariance of each part can be preserved to the case of the extrapolation, when because of additional photons, condition $p_a + p_b = p_c + p_d + k_1$ is not valid.

Basic construction elements

• KKMC/Tauola/Photos all have exact complete phase space under control. That is good news for systematic.

• Ambiguities require studies of matrix elements.

• However, as off-shelf results are often partly intergated and not immediate to separate into parts they can not be used without costly adaptations...

• Another problem. For consistency, often irrelevant at low energies some approximatrions were used: same treatment of hard γ^* and Z imposed $\gamma - \gamma$ box diagram approximations in `KKMC`.

• For `photos` MC it was phase space Jacobians matching of multibranch could not be better than order of ME applied. Expansion in curvature (Ricci tensor) must be consistent. Hard learned lesson.

• The path for ambiguities is clear but long and bumpy...

In previous slide I have show what is needed for some contributions.

Tests and low energies limitations

History:

- KORALB, KORALZ and predictions for many observable

- semi analytical and semi-analytical plus (structure functions calculations for selected observables, plus fixed order MC corrections).

- **Low Energies (potential issues):**

- Role of mass terms in radiative corrections. Important for radiative return configurations and box diagrams.

- radiative return logarithms and final state mass (phase space) effects.

- Pretabulation of electroweak corrections and what about $\alpha_{QED}(s)$, is it taken into account with sufficient granularity at low energies?

- Technical parameters may require different set-ups.

- Expertise of people, tests, man power, training .

Versions and their distributions

$\longrightarrow$ Published versions of KKMC-f77 and KKMC-C++

$\longrightarrow$ KKMC-F77 BASF2 of Belle collaboration validated $\alpha_{QED}(s)$ at 10 GeV CMS energies and below as well.

$\longrightarrow$ https://github.com/KrakowHEPSoft/KKMCee kkmc@uj.edu.pl

At present I am not involved in servicing this version. But I have it (after some difficulties) running. On the way I had encounter annoying problems due to some versions of `root` : old were ok, new were ok, intermediate were messing up event weights.

- Some tests are running fine, but many are waiting to be transferred from FORTRAN.

- Some known, and fixed issues, like `spin` entries in `HepMC3` seem not to be transmitted to distribution.

New applications: g-2

- If anomaly of $(g\text{-}2)_\mu$ is a reality then there should be an impact elsewhere too.
- New interactions are often proportional to the lepton mass.
- Then, why not in τ leptons?
- Also, why not anomalous electric/magnetic dipole moments?
- How this could manifest itself in observables?
- We have implemented such extra interactions first for `KKMC` MC at Belle 2 energies Phys.Rev.D 106 (2022) 11, 113010
- Then for `KKMC` at FCC energies 2407.17282
- Recently (2407.17282 and next week code semi public) for `TauSpinner` and LHC applications.

New applications: g-2

From g-2 to τ lepton decays

In collaboration with by **A. Korchin**†,*

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• **(1)** Anomalous dipole moment (g-2)$_\mu$ of interest $\rightarrow$ theory-experiment tensions.

• **(2)** To resolve: (i) measure better, (ii) calculate better; revise its input, (iii) discovery. experiments need new theory, then theory other experiments, which...

• **(3)** My experience goes with (ii), in particular:

\- **(4)** rad. return evts. for $\alpha_{QED}(s)$? predictions for $e^+e^- \rightarrow \pi^+\pi^-(l^+l^-)n\gamma$ need improvements. at work $\rightarrow$ man power, motivation from (g-2)$_\mu$ helps.

\- **(5)** **Also for (g-2) ambiguities**: help from $\tau \rightarrow \nu\ hadrons$for $\alpha_{QED}(s)$ too (after isospin rotation and optical theorem).

\- **(6)** If anomalous (g-2)$_\mu$, then electric magnetic dipole moments in τ's? • **(7)** I can not ignore kitchen for τ production and decay,
also how bremsstrahlung (i.e. QED) separates from hard interactions.

• Beyond Standard Model anomalous dipole moments may come from New Physics particles in loop corrections to $Z(\gamma) - \mu^+ - \mu^-$ vertex. If seen, that would bring fundamental change.

• Tension between theory predictions and experimental measurement for anomalous magnetic dipole moment (g-2)$_\mu$ is at the level of 5 σ : Fermilab Muon g-2 Experiment, Phys Rev Lett **131** (2023) 161802. If it will survive time test $\rightarrow$ change in our understanding of physics foundations.

• Measurements (high precision): a_μ(FNAL) = 116 592 055(24)$\cdot 10^{-11}$(0.20 ppm). New experimental world average: a_μ(exp) = 116 592 059(22)$\cdot 10^{-11}$ (0.19 ppm).

• For the theory side: Muon g-2 Theory Initiative: T. Aoyama et al., *The anomalous magnetic moment of the muon in the Standard Model*, Phys. Rep. 887, 1 (2020).

• QED multi loop calculations are impressive: T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio, Complete Tenth-Order QED Contribution to the Muon g-2, Phys. Rev. Lett. 109, 111808 (2012). But this is not the end of the story...

AMBIGUITIES is my life

New applications: g-2

I will address only some of them. Those where I have something to say.

Systematic of `KKMC` $e^+e^- \to \mu^+\mu^-(or\ hadrons)\ n\gamma$ and τ decay Monte Carlos, slide no. 12.

In particular of $e^+e^- \to \mu^+\mu^- n\gamma$, with hard γ's , and τ decay Monte Carlos spectral functions and what will change if final state is $\pi^+\pi^-$.

New applications: KKMC for $\pi^+\pi^-$

• Because of systematic ambiguities for g-2 interpretation, there is an interest to extend data pool for dispersion relations used in $\alpha_{QED}(s)$ evaluation. That means $e^+e^- \to hadrons$ total cross section as function of s.

• Belle data can be useful, the `KKMC` radiative return to small invariant masses of μ-pair can be used if...

$\to$ Muon mass has to be changed to pion mass.

$\to$ Its PDGid changed,

$\to$ Born level cross section re-weighted from fermion to scalars

$\to$ effect of intermediate resonances imprinted to the weight as well.

• How well it can work? Theoretical ambiguities will be larger than for μ.

• Anyway another helpful tool. Cross check with Phokara.

• Implementation prepared by Jadach is not easy to recover.

New applications: KKMC for $\pi^+\pi^-$

• Bremsstrahlung and its virtual counter-part happen at smaller energies than hard interaction e.g. of $\gamma^* \to \tau^+\tau^-$.

• That sounds simple, but requires effort to separate. Without proper evaluations of Standard Model predictions and their ambiguities:

— no input for $\alpha_{QED}(s)$ from radiative return events

• If New Physics would not separate consequences would be less severe: that would be New Physics anyway, but its nature would be more difficult to interpret.

• To do that one has to keep under control issues of factorization:

— scale of QED bremsstrahlung (with its virtual corrections) differ from scale of hard process.

• That is useful for intuition, but if ambiguities are to be taken into account, serious work on properties of spin amplitudes are needed.

• Work on semi inclusive quantities, is not enough. Thanks to the efforts, we can advocate `Mustraal` frame to separate hard and bremsstrahlung (parton level) interactions parts of phase-space and amplitudes.

• I will cover this topic, at a side of discussion on: turning $e^+e^- \to \mu^+\mu^- n\gamma$ Monte Carlo into one of $e^+e^- \to \pi^+\pi^- n\gamma$. It is of importance in itself though.

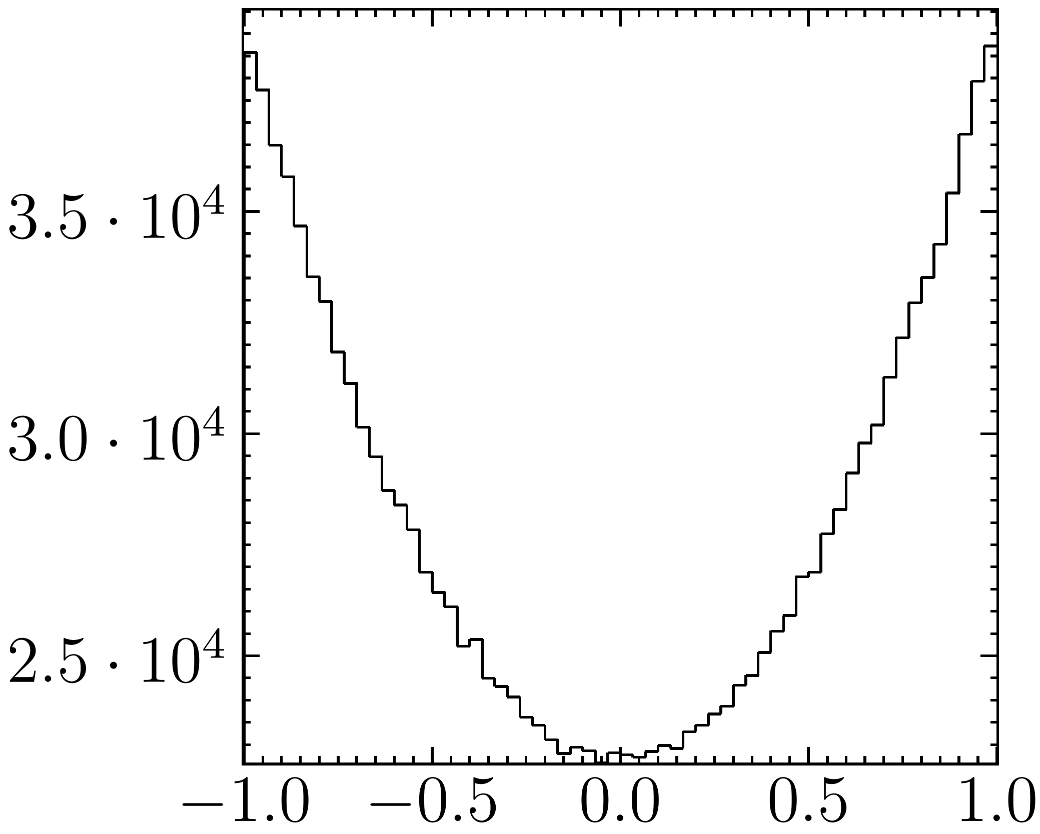
New applications: KKMC for $\pi^+\pi^-$

- **Promising path $\rightarrow$** use `Mustraal` frame. Minimize bias from lack of bremsstrahlung amplitudes and virtual corrections for π's. At the time of LEP 1 Monte Carlo (1982) work, amplitudes for $e^+e^- \rightarrow \mu^+\mu^-\gamma$ were carefully studied.
- It was found (no approximation): fully differential cross section could be written in form of factorization of photon emission factor and corresponding phase space term of photon energy and its $\theta\phi$ angles, times the factor of the same structure as Born.
- Sum of two expressions of slightly different frames orientation was necessary.
- Formed incoherent sum, excellent for MOnte Carlo implementation.
- There is a theorem by Rondald Kleiss that such separation is quite universal and got first correction from α^2 terms (without any enhancement).
- We have collected numerous tests for that in context of spherical harmonics used nowdays in LHC data interpretation.
- The checks were for QCD and up to second order (two parton emissions).
- No need to repeat that work for $\pi^+\pi^-$: properties originate from Lorentz group structure: its layers with respect to rotation sub-group.

New applications: KKMC for $\pi^+\pi^-$

Figure 2: Mustraal frame $\cos\theta$ distributions for muons. Only hard bremsstrahlung events taken.

Test level 0 : No deformations with respect to Born level shape.

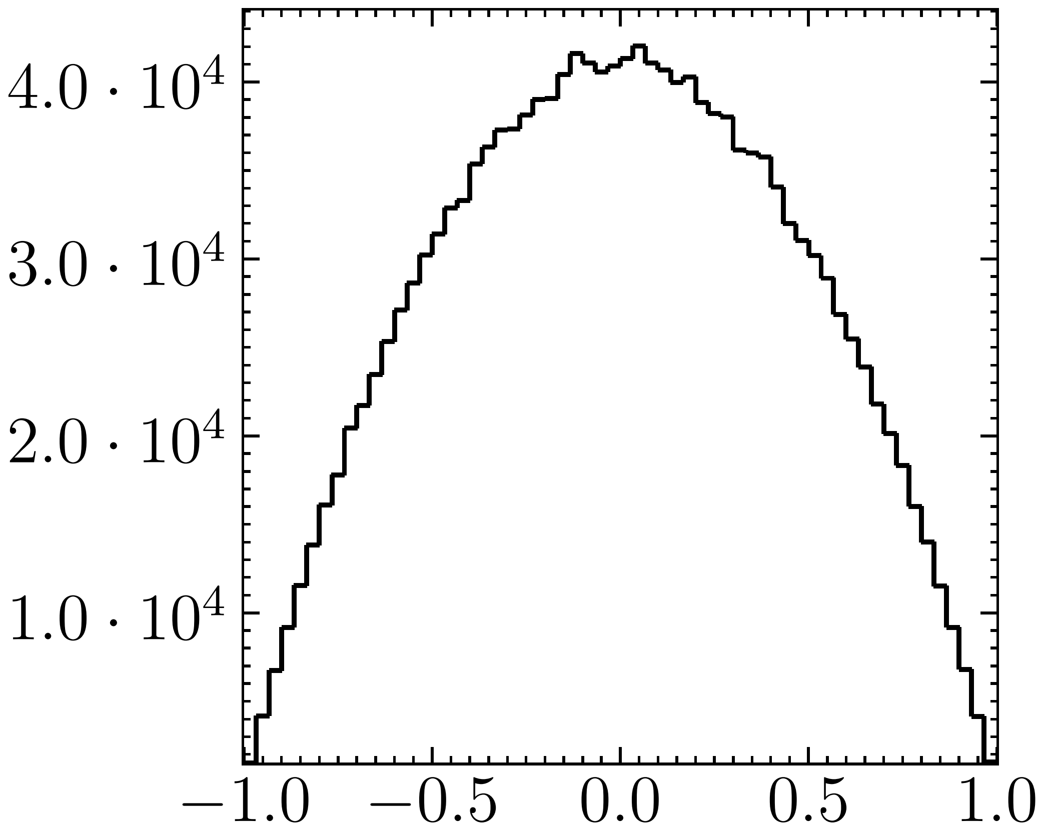


New applications: KKMC for $\pi^+\pi^-$

Figure 3: `Mustraal` frame $\cos\theta$ distributions for pions. re-weighting from muon sample.

Only hard bremsstrahlung events taken. Reweighting of BASF2-lib events with user code.

Test level 0 : No deformations with respect to Born level shape.



New applications: KKMC for $\pi^+\pi^-$

- **But what about FSR and IFI interference, for $\pi^+\pi^-$ final states?**
- Message from scalar QED is not very encouraging. amplitudes for $\gamma^* \to \pi^+\pi^-\gamma$ are complicated (0802.2182) have their own enhancements, which **DO NOT** match QED. Even QCD results (0802.2182) look better.
- Hope: feed back (fits, form factors) from experiments like BESIII may settle the matter.
- Not as straighforward and theoretical considerations inescapable?
- **GOOD:** eikonal part of amplitdue separates out. Complete first order contribution to final state bremsstrahlung: $\gamma^* \to \pi^+\pi^-\gamma$, can be unambiguously added through event reweighting.
- **SURPRIZE:** this weight is becoming large when $\pi^+\pi^-$ pair invariant mass is large. No problem for `Photos`, but possibly points to ambiguity and in particular in interference with ISR.
- **BAD:** Radiative return ambiguity? Interference with ISR may be larger than I would bet before Pisa.

Outlook

- The purpose of my talk was to push some ideas forward and what is needed for that.
- It was not disciplined talk. Sorry for that.
- My aim was to underline directions, which in my opinion, are worth to follow.
- I have sketched also what are the main aspects of the activities now.
- I hope it may be useful.
- **I promise to contribute,** I hope to complete training of new bunch of contributors too.
- I hope they will then find path to continue, stay in the field etc. etc.
- Thanks for listening.

ps. In Cracow it is even colder than here. Last week we had +28 degrees.