Workshop on Radiative Corrections and Monte Carlo simulations for electron-positron collisions

A mule never stops
– future plans for McMule –

Sara Gündogdu
for the McMule team

What are the challenges?

Established in McMule

$2 \to 2$ QED NNLO
$2 \to 3$ QED NLO [David's talk]
$2 \to \pi\pi\gamma$ NLO ISC [Sophie's talk]

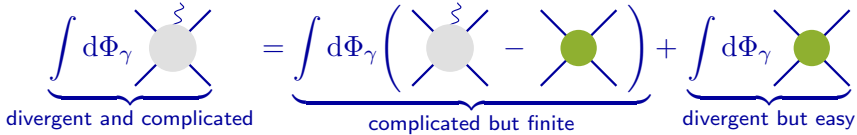
$\Longrightarrow$

McMule goal

N^3LO (MuonE)
NNLO
NLO mixed & NNLO ISC

Monte **C**arlo for **MU**ons and other **LE**ptons

- integrator (generator WIP) for fixed-order QED up to NNLO
- use QCD methods: FKS$^\ell$ subtraction with massive fermions



- challenge virtual amplitudes with $m \neq 0 \implies$ massification (photonic)
- challenge numerical instabilities $\implies$ next-to-soft stabilisation + **O**pen**L**oops

Monte **C**arlo for **MU**ons and other **LE**ptons

- integrator (generator WIP) for fixed-order QED up to NNLO
- use QCD methods: FKS$^\ell$ subtraction with massive fermions
- challenge virtual amplitudes with $m \neq 0 \implies$ massification (photonic)

$$\mathcal{A}(m) = \left(\prod_j \sqrt{Z(m)}\right) \times \mathcal{A}(m=0) + \mathcal{O}(m) \quad \text{iff} \quad m^2 \ll \text{all other scales}$$

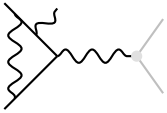
- challenge numerical instabilities $\implies$ next-to-soft stabilisation + **O**pen**L**oops

process#	experiment	physics motivation	order
$e\mu \to e\mu$	MUonE	HVP to $(g-2)_\mu$	NNLO
$\ell \mathrm{N} \to \ell \mathrm{N}$	P2, Muse, Prad, QWeak, ...	proton radius and weak charge	NNLO$(-)$
$e\nu \to e\nu$	DUNE	flux & $\sin^2\theta_W$	NNLO$-$
$e^-e^- \to e^-e^-$	Prad	normalisation	NNLO
	MOLLER, ...	$\sin^2\theta_W$ at low Q^2	
$e^+e^- \to e^+e^-$	any e^+e^- collider	luminosity measurement	NNLO
$ee \to \gamma^*$			NNLO
$ee \to \ell\ell$	CMD+SND, BES, KLOE, ...	R-ratio	NNLO+
	Belle	τ properties & $\sin^2\theta_W$	
$ee \to \pi\pi$	CMD+SND, BES, KLOE, ...	R-ratio	NLO+
$ee \to \gamma\gamma$	KLOE	dark searches	NNLO$-$
	any e^+e^- collider	luminosity measurement	
$\mu \to \nu\bar{\nu}e$	MEG, Mu3e, Pioneer, Mu2e	ALP searches	NNLO+
	DUNE	beam-line profiling	
$\mu \to \nu\bar{\nu}eee$	Mu3e	background	NLO

process#	experiment	physics motivation	order
$e\mu \to e\mu$	MUonE	HVP to $(g-2)_\mu$	NNLO
$\ell\mathrm{N} \to \ell\mathrm{N}$	P2, Muse, Prad, QWeak, ...	proton radius and weak charge	NNLO$(-)$
$e\nu \to e\nu$	DUNE	flux & $\sin^2\theta_W$	NNLO$-$
$e^-e^- \to e^-e^-$	Prad	normalisation	NNLO
	MOLLER, ...	$\sin^2\theta_W$ at low Q^2	
$e^+e^- \to e^+e^-$	any e^+e^- collider	luminosity measurement	NNLO
$ee \to \gamma^*\gamma$			NLO$+$
$ee \to \ell\ell\gamma$	CMD+SND, BES, KLOE, ...	R-ratio	NLO$+$
	Belle	τ properties & $\sin^2\theta_W$	
$ee \to \pi\pi\gamma$	CMD+SND, BES, KLOE, ...	R-ratio	LO$+$
$ee \to \gamma\gamma$	KLOE	dark searches	NNLO$-$
	any e^+e^- collider	luminosity measurement	
$\mu \to \nu\bar{\nu}e$	MEG, Mu3e, Pioneer, Mu2e	ALP searches	NNLO$+$
	DUNE	beam-line profiling	
$\mu \to \nu\bar{\nu}eee$	Mu3e	background	NLO

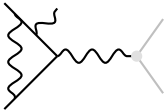
$ee \to \gamma\gamma^*$@ NLO

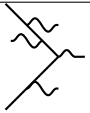
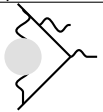
- universal framework for arbitrary currents X $\in \{\pi, {}^{12}C, p, {}^{2}H, \dots\}$
- full mass dependence $ee \to XX$



$ee \to \gamma\gamma^*$@ NLO

- universal framework for arbitrary currents $X \in \{\pi, {}^{12}C, p, {}^{2}H, \dots\}$
- full mass dependence $ee \to XX$

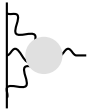


	$ee \to \gamma\gamma^*$ @ NNLO	methods & challenges
RR		OL ✓ NTS (?) B/BES scenario (?)
RV		OL ✓ NTS ✓
VV		massification ✓ [Badger et al 23] KLOE SA hard-collinear (?)
VP		Disperon QED [Sophie's talk]

next step [$\sim$ 2026+] :: $ee \to \gamma^*$ @ N^{3}LO $\supset$ $ee \to \gamma\gamma^*$ @ NNLO

[Fael et al 22]

[Badger et al 23]
+ massification
+ jettification

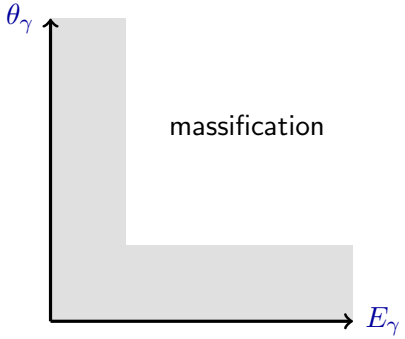


light-by-light, ...

RVV matrix element needs **massification**

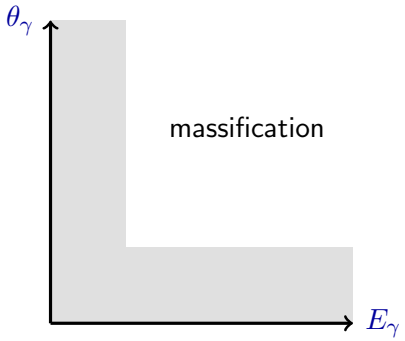
$$\mathcal{M}_n(m) \xrightarrow{m\to 0} \mathcal{M}_n(0) \times Z \times Z \qquad \mathcal{M}^{(2)}_{n+1} \sim \frac{1}{E_\gamma^2}\frac{1}{(1-\beta\cos\theta_\gamma)}$$

- ($m^2 \ll$ all other scales) not valid everywhere



RVV matrix element needs **massification**

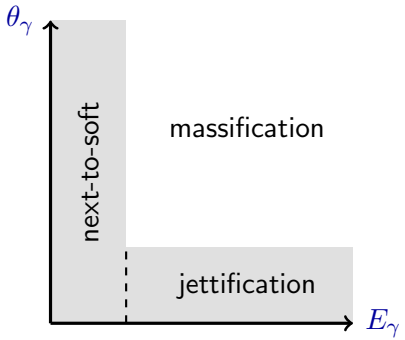
$$\mathcal{M}_n(m) \xrightarrow{m\to 0} \mathcal{M}_n(0) \times Z \times Z \qquad \mathcal{M}^{(2)}_{n+1} \sim \frac{1}{E_\gamma^2}\frac{1}{(1-\beta\cos\theta_\gamma)}$$



- ($m^2 \ll$ all other scales) not valid everywhere
- detected photon ($ee \to \gamma\gamma^*$ @ NNLO) :
  region excluded by cuts : ✓
  Kloe SA scenario region included : ✗

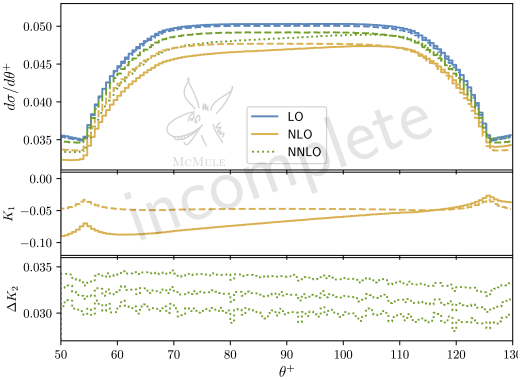
RVV matrix element needs **massification**

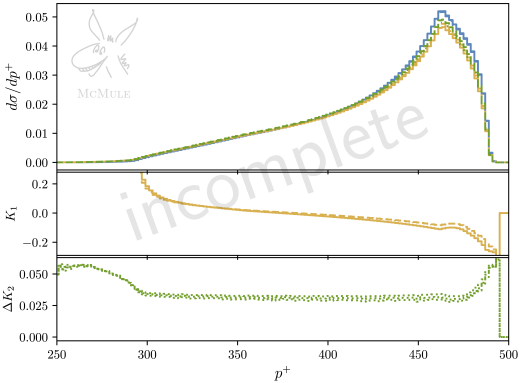
$$\mathcal{M}_n(m) \xrightarrow{m\to 0} \mathcal{M}_n(0) \times Z \times Z \qquad \mathcal{M}^{(2)}_{n+1} \sim \frac{1}{E_\gamma^2}\frac{1}{(1-\beta\cos\theta_\gamma)}$$



- ($m^2 \ll$ all other scales) not valid everywhere
- detected photon ($ee \to \gamma\gamma^*$ @ NNLO) :
  region excluded by cuts : ✓
  Kloe SA scenario region included : ✗
- inclusive process ($ee \to \gamma^*$ @ N^{3}LO):
  next-to-soft
  jettification: massive J unknown at 2 loop

$$\mathcal{M}_{n+1}(m) \xrightarrow[\theta_\gamma\to 0]{m\to 0} \mathcal{M}_n(0) \times Z \times J$$





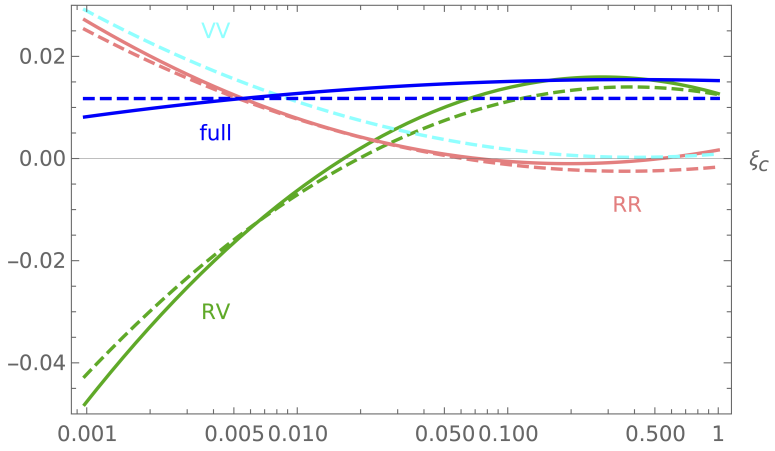
$\sqrt{s} = 1.02\,\text{GeV}$;

$50^\circ \leq \theta^\pm \leq 130^\circ$;

$|p_z^\pm| > 90\,\text{MeV} \vee |p_\perp^\pm| > 160\,\text{MeV}$;

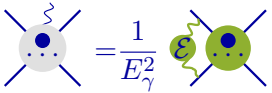
$\theta_{\tilde{\gamma}} \leq 15^\circ \vee \theta_{\tilde{\gamma}} \geq 165^\circ$;

$0.35\,\text{GeV}^2 \leq M_{\mu\mu}^2 \leq 0.95\,\text{GeV}^2$

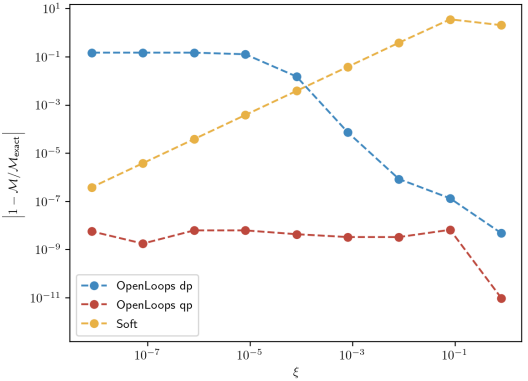

VV
full
RR
RV
0.02
0.00
−0.02
−0.04
0.001
0.005
0.010
0.050
0.100
0.500
1
ξ_C

matrix elements with real emission need **next-to-soft stabilisation**

current state :: NTS for RV $\Longrightarrow$ automatized for arbitrary process

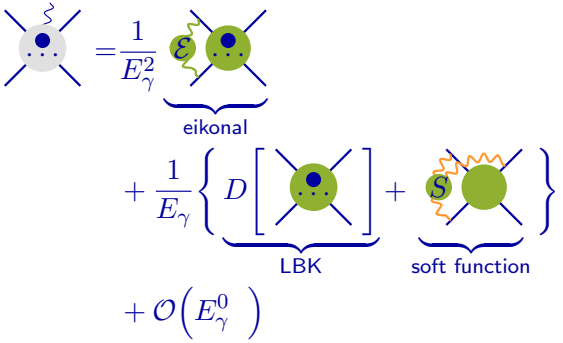


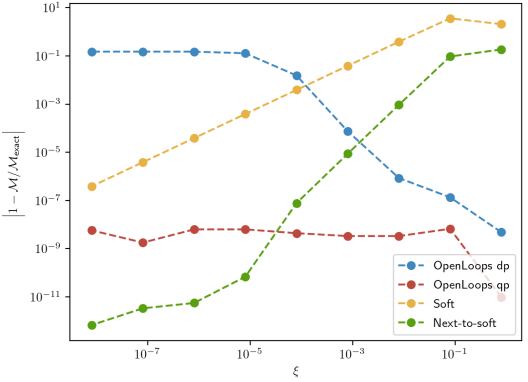
$$+ \mathcal{O}\left(E_\gamma^{-1}\right)$$



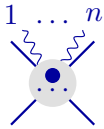
matrix elements with real emission need **next-to-soft stabilisation**

current state :: NTS for RV $\Longrightarrow$ automatized for arbitrary process





McMule goal : NTS for RRR, RRV and RVV [Engel, 24]



$$= \frac{1}{E_\gamma^{2n}} \prod_{a=1}^{n} \mathcal{E}_{k_a}$$

eikonal

$k_1 \sim k_2 \sim \cdots \sim k_n$

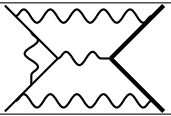
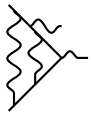
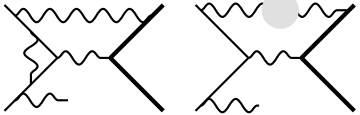
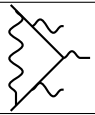
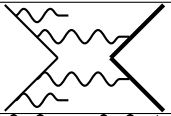
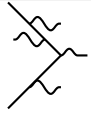
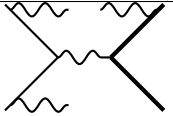
$$+ \frac{1}{E_\gamma^{2n-1}} \left\{ \sum_{a=1}^{n} \prod_{c \neq a} \mathcal{E}_{k_c} \left(D_{k_a} \left[\right] + S_{k_a} \right) \right.$$

LBK

soft function

$$\left. + \sum_{a,b=1}^{n} \prod_{c \neq a,b} \mathcal{E}_{k_c} G_{k_a}^{k_b} \right\} + \mathcal{O}\left(E_\gamma^{-2n+2}\right)$$

universal function

	$ee \to \gamma^*$	$ee \to \mu\mu$ ($e\mu \to e\mu$ MuonE)	challenges
VVV			massification & prayers
VVR			from $pp \to 2j + \gamma$ [Badger et al 23]+massification Disperon QED, NTS, jettification
VRR			OL, NTS
RRR			NTS

Ideas for the future

- event generation
- additional ee pair
- McMule @ higher energies ::
  numerical instability for real-real in $ee \to \mu\mu\gamma$ @ B/BES-like $\implies$ collinear subtraction?
- YFS/LBK shower - approximate higher orders
- electroweak
- polarisation
- ...

f.l.t.r.: S.Kollatzsch (Zurich & PSI), A.Signer (Zurich & PSI), V.Sharkovska (Zurich & Mainz), S.Gündogdu (Zurich & PSI), D.Moreno (PSI), A.Coutinho (IFIC), Y.Ulrich (Liverpool), D.Radic (Zurich & PSI), L.Naterop (Zurich & PSI), M.Rocco (Turin)
not shown: F.Hagelstein (Mainz), N.Schalch (Oxford), P.Banerjee (Cosenza), M.Ronchi (Mainz), Y.Fang (ETH), P.Wahlen (ETH), R.Krolzig (UZH)

McMule

mule-tools.gitlab.io